

Dipoles in the sky

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Accepted 2012 September 2. Received 2012 August 28; in original form 2012 June 20

ABSTRACT

We perform observational tests of statistical isotropy using data from large-scale-structure surveys spanning a wide range of wavelengths. Using data from Two Micron All Sky Survey (2MASS), 2MASS Redshift Survey (2MRS), NRAO VLA Sky Survey (NVSS) galaxies, and Burst And Transient Source Experiment (BATSE) gamma-ray bursts, we constrain the amplitude and direction of dipolar modulations in the number count of sources projected along the line of sight. We pay particular attention to the treatment of systematic errors and selection effects, and carefully distinguish between different sources of dipole signal previously considered in the literature. Dipole signals detected in these surveys are consistent with the standard, statistically isotropic expectation, except for the NVSS result, which is likely biased by remaining systematics in the data. We place constraints on the amplitude of any intrinsic dipole driven by novel physics in the early universe.

Key words: large-scale structure of Universe.

1 INTRODUCTION

The cosmological principle holds that the universe is homogeneous and isotropic on its largest observable scales. While the cosmological principle is a crucial ingredient in obtaining many important results in quantitative cosmology, there is no fundamental reason why our universe *must* obey it. The most general aim of the research we present here has been to directly test the cosmological principle using data from recent astrophysical observations.

Observations do bear out that the cosmological principle is a reasonable approximation for most purposes. However, few rigorous observational tests have been applied to test homogeneity and isotropy. The work presented here is directed towards performing tests of the statistical isotropy of the universe using large-scale structure (LSS). The goal of this work is, fundamentally, to bring statistical isotropy more fully out of the realm of assumption and into the realm of observation. Any observed violations of statistical isotropy could have far-reaching implications for our understanding of the universe's earliest moments, and violations of isotropy would also invalidate basic assumptions that serve as prerequisites to typical methods of data analysis in observational cosmology.

To define statistical isotropy, consider a fluctuating field on the sky $T(\hat{n})$ (the same arguments will apply for any field, including the CMB temperature field, the galaxy density field, etc.). The field is statistically isotropic if the two-point correlation function depends *only* on the separation between points:

$$\langle T(\hat{n})T(\hat{n}') \rangle = C(\hat{n} \cdot \hat{n}') \quad (1)$$

while in the case where statistical isotropy is violated, the right-hand side would read $C(\hat{n}, \hat{n}')$.

Alternatively, we could expand the field in spherical harmonics $T(\hat{n}) \equiv \sum_{\ell=0}^{\infty} T_{\ell} \equiv \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\hat{n})$, where the $a_{\ell m}$ are the coefficients of the expansion. If statistical isotropy is assumed, the angular power spectrum C_{ℓ} may be defined via $\langle a_{\ell m}^* a_{\ell' m'} \rangle = C_{\ell} \delta_{\ell \ell'} \delta_{m m'}$, where C_{ℓ} does not depend on the rotational degrees of freedom m . Hence, it is meaningful to calculate C_{ℓ} by averaging over the $(2\ell + 1)$ (on a full sky) ‘samples’ corresponding to the $(2\ell + 1)$ values of m for each ℓ . However, without the assumption of statistical isotropy, we have in general

$$\langle a_{\ell m}^* a_{\ell' m'} \rangle = \tilde{C}_{\ell m \ell' m'}, \quad (2)$$

which is much more difficult to measure than C_{ℓ} since we get only one sample of $\tilde{C}_{\ell m \ell' m'}$ for each (ℓ, m, ℓ', m') in our universe. This is a big reason why statistical isotropy is such a crucial assumption in cosmology: it is much easier to work with C_{ℓ} than the much more complicated quantity above.

1.1 Point of departure: large-scale structure as probe of CMB anomalies

While measurements of the angular power spectrum of the CMB by the *Wilkinson Microwave Anisotropy Probe* (WMAP) experiment have provided strong support for the inflationary hot big bang model and allowed for unprecedentedly precise determination of cosmological parameters, these successes have come along with various unexpected ‘anomalies’ in the data (for reviews, see Copi et al. 2010; Bennett et al. 2011).

There are already tantalizing hints of violations of statistical isotropy in CMB data; in fact, several observed anomalies,

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especially those indicating correlations between patterns in the CMB of (apparently) cosmological origin (e.g. the quadrupole and octopole) and the geometry and motion of the Solar system, can be understood as indications that statistical isotropy fails to hold. In addition, the southern ecliptic hemisphere has significantly more power than the northern ecliptic hemisphere on scales of about 3° and larger (multipoles $\lesssim 60$) (Hoftuft et al. 2009). This so-called ‘hemispherical power anomaly’ is a dipole modulation (see the beginning of Section 3 for precise discussion of what this entails mathematically) of CMB power [direction in Galactic coordinates is $(l, b) = (224^\circ, -22^\circ)$] that is completely distinct from the measured CMB dipole C_1 (which is due to our motion with respect to the CMB rest frame), and is yet another possible indication of breaking of statistical isotropy in the CMB.¹ Finally, the low power at large angles in $C(\theta)$ may itself be an indication that statistical isotropy is violated: it seems that a conspiracy of low- ℓ multipoles in the angular power spectrum C_ℓ is responsible for creating the suppressed $C(\theta)$ (Copi et al. 2009), and correlations between different multipoles could be the result of a lack of statistical isotropy.

1.2 Goals of this work

We take these considerations as general motivation for the work performed in the rest of this paper, which concerns testing statistical isotropy with LSS, and will analyse the specific issue of dipole patterns in various surveys being used as probes of the statistical isotropy of the universe. This work also complements the (surprisingly few!) studies of tests of statistical isotropy with current or future LSS observations (Ellis & Baldwin 1984; Hirata 2009; Gibelyou, Huterer & Fang 2010; Pullen & Hirata 2010; Zunckel, Huterer & Starkman 2011).

The paper is organized as follows. In Section 2, we point out the various effects that are expected to contribute to a dipole signal in a LSS survey. In Section 3, we outline the formalism used in this paper to detect such dipole signals. We then apply that formalism to several surveys: the 2MASS Redshift Survey (2MRS; Section 4), the Two Micron All Sky Survey (2MASS) as a whole (Section 5), the Burst And Transient Source Experiment (BATSE) of the *Compton Gamma-Ray Observatory* (CGRO; Section 7) and the NRAO VLA Sky Survey (NVSS; Section 8). In Section 6, we also examine issues surrounding searching for dipole signals in high-redshift objects and review work that has searched for dipoles in X-ray surveys. We conclude in Section 9, summarizing our results in Table 8 in that section.

2 TYPES OF DIPOLES: SPECIFIC THEORETICAL CONSIDERATIONS

It is completely expected that a dipole will be present in any survey of objects that trace LSS. Both of the following effects contribute to the dipole: (a) there are local anisotropies since the universe is not homogeneous and isotropic except on its very largest scales and (b) the Earth has a total motion relative to the LSS rest frame

¹ However, see Hanson & Lewis (2009) for the updated result that the significance of the hemispherical power anomaly decreases when smaller scales are taken into account: previous analyses had been done for $\ell \lesssim 60$; when analysis is extended out to $\ell \sim 500$, the effect becomes less than 3σ -anomalous. The fact that greater resolution reduces the significance of the signal calls into question the authenticity of the signal as a genuine cosmological effect.

that is the sum of several vector contributions (the Earth moves around the Sun, the Sun moves around the centre of the Milky Way, the Milky Way moves with respect to the Local Group barycentre and the Local Group barycentre moves with respect to the structure around it and, ultimately, the LSS rest frame). That motion produces dipole anisotropy due to two effects, the Doppler effect and relativistic aberration of angles (Itoh, Yahata & Takada 2010; see Section 2.3 for mathematical details). The Doppler effect is relevant because it changes how magnitude varies with frequency, and since LSS surveys invariably operate within limited frequency ranges, the Doppler effect may shift certain objects into or out of a magnitude-limited sample. Since frequencies will increase in the direction of motion and decrease in the opposite direction, this produces a small dipole in the number of objects detected. Meanwhile, relativistic aberration causes the measured positions of galaxies to be displaced towards our direction of motion. This effect is of the order of $v/c \sim 10^{-3}$, relevant for our purposes.

We expect that as we go from smaller to larger survey volumes, the measured value of the dipole amplitude should converge to that of the CMB dipole. This is because with larger survey volumes, we average over more and more structure, and the universe approaches homogeneity and isotropy. Any dipole left over should be due only to our motion, a kinematic dipole (with amplitude on the order of 10^{-3}) just as in the CMB. There are several reasons why the dipole might *not* converge to that of the CMB.

(i) The rest frame of the CMB may not be the same as the rest frame of the LSS: novel horizon-scale physics (explored below) could induce a relative velocity between the CMB and LSS, so that galaxies would have a non-zero average streaming velocity with respect to the CMB rest frame.

(ii) There is also the possibility that there is genuinely more mass (and therefore more galaxies/objects that trace the mass distribution) in one direction, corresponding to modulation of primordial curvature perturbations due to the physics of inflation. For example, isocurvature perturbations can produce such an effect (and explain the CMB hemispherical power anomaly; Langlois 1996; Langlois & Piran 1996; Erickcek, Kamionkowski & Carroll 2008b; Erickcek, Hirata & Kamionkowski 2009).

Careful measurement of dipoles in various surveys, such as those we perform here, help zone in on these possibilities, which correspond to a violation of statistical isotropy.

The rest of this section will flesh out the details of the discussion in the preceding two paragraphs.

2.1 Flux-weighted dipole versus 2D-projected dipole

One very commonly computed type of dipole is not, strictly speaking, a dipole at all, but is frequently referred to as such. This is the flux-weighted ‘dipole’, where instead of calculating a (genuine) dipole based only on the 2D-projected positions of objects on the sky, some radial information is preserved by weighting each object by the flux we receive from it. We follow Bilicki et al. (2011) in the following explanation of how the flux-weighted dipole is calculated.

The flux-weighted dipole, as typically computed, is a measure of the acceleration due to gravity on the Local Group. In linear theory, the peculiar velocity $\mathbf{v}(\mathbf{r})$ at position \mathbf{r} is proportional to the peculiar acceleration vector $\mathbf{g}(\mathbf{r})$ induced by the matter distribution around position \mathbf{r} (Erdogdu et al. 2006; Bilicki et al. 2011):

$$\mathbf{v}(\mathbf{r}) = \frac{H_0 f(\Omega_M)}{4\pi G \bar{\rho}} \mathbf{g}(\mathbf{r}) = \frac{2f(\Omega_M)}{3H_0 \Omega_M} \mathbf{g}(\mathbf{r}). \quad (3)$$

Here $H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$ is the Hubble constant, Ω_M is the matter density divided by the critical density and $f(\Omega_M) \equiv (d \ln D / d \ln z)|_{z=0}$ (where D is the growth factor). In the Lambda cold dark matter (Λ CDM) model, the factor $f(\Omega_M) \approx \Omega_M^{0.55}$ and is only weakly dependent on the cosmological constant (Lahav et al. 1991). The acceleration vector itself is given by

$$\mathbf{g}(\mathbf{r}) = G\bar{\rho} \int \delta_M(\mathbf{r}') \frac{\mathbf{r}' - \mathbf{r}}{|\mathbf{r}' - \mathbf{r}|^3} d^3\mathbf{r}', \quad (4)$$

where $\delta_M(\mathbf{r}) = [\rho_M(\mathbf{r}) - \bar{\rho}] / \bar{\rho}$ is the density contrast of the mass perturbations at the point \mathbf{r} ; note that ρ_g is the mass density of galaxies, and b is the bias factor that relates mass density of galaxies to that of matter, $b \equiv \rho_g / \rho_M$ (assuming constant, scale- and time-independent bias). The bias is usually packaged with the factor $f(\Omega_M)$ into the parameter $\beta \equiv f(\Omega_M)/b$. Comparing equations (3) and (4), we get the proportionality valid in linear theory:

$$\mathbf{v} \propto \beta \mathbf{g}. \quad (5)$$

Comparison of the peculiar velocity \mathbf{v} (determined from either the LSS surveys or using the CMB dipole) and acceleration \mathbf{g} of the Local Group serves as a tool to estimate the β parameter. Independent knowledge of biasing allows one to estimate the cosmological matter density Ω_M . The programme of measuring the matter density in this way has been ongoing for over three decades (e.g. Yahil, Sandage & Tammann 1980; Davis & Huchra 1982; Yahil, Walker & Rowan-Robinson 1986; Lahav, Lynden-Bell & Rowan-Robinson 1988; Hudson 1993; Lauer & Postman 1994; Lavaux et al. 2010; Davis et al. 2011; Nusser & Davis 2011; Nusser, Branchini & Davis 2011).

The challenging part in this procedure is evaluating the acceleration in equation (4). This is where the flux-weighted-dipole approach comes in, where position vectors of the objects in a survey are weighted according to their fluxes (used as a rough proxy for mass since both gravity and flux go as $1/r^2$), and added together. By preserving some radial information in this way, the flux-weighted dipole allows one to obtain a measure of the direction and strength of the acceleration of the Local Group due to the Newtonian gravitational attraction from objects in the survey. The flux-weighted dipole from 2MASS has previously been found to be in the direction $(l, b) = (264^\circ.5, 43^\circ.5) \pm (2^\circ.0, 4^\circ.0)$ [Maller et al. 2003; note the rather serious discrepancy between the published result and the arXiv version, and see Bilicki et al. (2011) for detailed discussion of convergence of the flux-weighted dipole and possible shortcomings of Maller et al.]; the flux-weighted dipole from 2MRS is in the direction $(l, b) = (251^\circ, 38^\circ)$ (Erdogdu et al. 2006) in the Local Group frame, or $(245^\circ, 39^\circ)$ in the CMB frame.

Just to be clear, we do *not* consider further the flux-weighted dipole in this work; we henceforth study the 2D-projected dipole described below.

2.2 2D-projected dipole: local-structure dipole

For the rest of this section, we focus on what we term the 2D-projected dipole, which is the quantity that is usually indicated by the isolated use of the word ‘dipole’. This quantity relies on objects at any given redshift being projected on the celestial sphere (hence ‘2D-projected’) with no weighting scheme.

For a survey with very large (hundreds of Mpc to Gpc scale) volume, the universe is at least close to homogeneous and isotropic on the scales relevant for the survey. We naturally expect that any dipole signal in such a large-volume survey will be strongly suppressed. However, on much smaller scales, where the universe is not

at all homogeneous and isotropic, dipole signals should naturally emerge in any survey of objects that trace LSS at all, and certainly in any galaxy survey. To take a particularly simple example, there is a large dipole in the galaxy distribution if we survey only objects within the Local Group.

However, even if a survey encompasses structure on scales of tens of Mpc, we fully expect that given the non-uniformity of nearby structure, there should be a dipole component in the pattern of galaxies observed on the sky. This dipole component will be strongest for the smallest surveys, and should die away monotonically (at least in statistical average) for larger and larger surveys. The effect turns out to be of the order of 10^{-1} for scales of tens to a couple of hundreds of Mpc, meaning that the fluctuations (contributing to the dipole) in the number of galaxies, as a function of position on the sky, are of the order of 1/10 the size of the mean number of galaxies across the sky.

More rigorously, we make predictions for the full angular power spectrum C_ℓ of LSS as a function of maximum redshift of a survey. The angular power spectrum of density fluctuations of haloes is usually expressed within the Limber approximation, where the contribution of modes parallel to the line of sight is ignored. In this approximation, the angular power spectrum is given by

$$C_\ell = \frac{2\pi^2}{\ell^3} \int_0^\infty dz \frac{W^2(z)}{r(z)^2 H(z)} \Delta^2 \left(k = \frac{\ell}{r(z)}, z \right), \quad (6)$$

where $\Delta^2(k) \equiv k^3 P(k) / (2\pi^2)$ is the dimensionless power spectrum, $r(z)$ is the comoving distance and $H(z)$ is the Hubble parameter. The weight $W(z)$ is given by

$$W(z) = \frac{b(z) \frac{dN}{dz}}{\int_{z_{\min}}^{z_{\max}} \left(\frac{dN}{dz} \right) dz}, \quad (7)$$

where z_{\min} and z_{\max} are the lower and upper end of the redshift range, and $dN/dz(z)$ is the number of galaxies per unit redshift.² We adopt the tabulated values, or else functional form, of $N(z)$ directly from the respective surveys that we study.

However, we are interested in the dipole $\ell = 1$ where the Limber approximation is not accurate anymore (it is accurate at $\ell \gtrsim 10$); see Fig. 1. Therefore, we adopt the exact expression for the power spectrum; using notation from (e.g.) Hearin, Gibelyou & Zentner (2011), this is

$$C_\ell = 4\pi \int_0^\infty d \ln k \Delta^2(k, z=0) I^2(k), \quad (8)$$

$$I(k) \equiv \int_0^\infty dz W(z) D(z) j_\ell(k\chi(z)), \quad (9)$$

where $\chi(r)$ is the radial distance, and $\chi(z) = r(z)$ in a flat universe, the case that we consider. Here $D(z)$ is the linear growth function of density fluctuations, so that $\delta(z) = D(z)\delta(0)$, where $D(0) = 1$. Note that, over the shallow range for 2MASS we can assume that $b(z)$ is constant, and factor it outside of equations (8) and (9), but over the much deeper range for NVSS the bias may vary with redshift, and we adopt the expression for $W(z)$ from Ho et al. (2008) that implicitly integrates bias and number density as per equation (7). This is explained in detail in Section 8.2.

² Note that a sometimes-used alternative definition of $n(z)$ refers to the spatial density of galaxies (e.g. Hu & Jain 2004); it is related to the quantity we use via $dN/dz = n(z) \Omega r^2(z) / H(z)$, where Ω is the solid angle spanned by the survey, and r and H are the comoving distance and Hubble parameter, respectively. Note also that our $W(z)$ is equivalent to the quantity $f(z)$ from Ho et al. (2008).

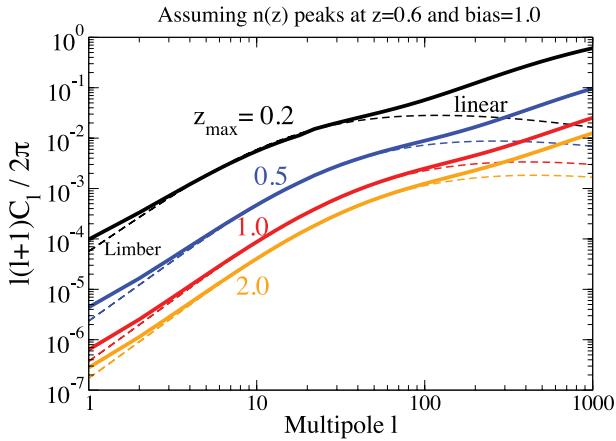


Figure 1. A plot of the angular power spectrum C_ℓ predicted for a galaxy survey with a peak in the galaxy redshift distribution at $z = 0.6$ and the given maximum depth z_{max} . In the remainder of this paper we will focus attention on the dipole, $\ell = 1$. For this particular redshift distribution, the local-structure dipole becomes subdominant to the kinematic dipole for around $z_{\text{max}} \sim 1.0$. For higher z_{max} , we should get convergence to the kinematic dipole plus any intrinsic dipole that might be present. The dotted curves correspond to the power spectrum within the Limber approximation (low ℓ) and assuming linearity (high ℓ); the solid curves correspond to the more accurate set of assumptions where the Limber approximation is relaxed and non-linearity is taken into account.

To produce the fiducial theoretical predictions, we consider the standard cosmological model with the following parameter values: matter density relative to critical $\Omega_M = 0.25$, equation of state parameter $w = -1$, spectral index $n = 0.96$ and amplitude of the matter power spectrum, $\ln A$, where $A = 2.3 \times 10^{-9}$ (corresponding to $\sigma_8 = 0.8$) defined at scale $k = 0.002 \text{ Mpc}^{-1}$. The power spectrum $\Delta^2(k, z) \equiv k^3 P(k)/(2\pi^2)$ is calculated using the transfer function output by CAMB. We do not vary the values of cosmological parameters since they are measured to sufficient accuracy that any shifts in predicted dipole amplitude that could occur due to realistic changes in cosmological parameters are tiny in comparison with cosmic variance given the finite sky coverage and relative shallowness of the surveys we employ (as we have explicitly verified).

The power spectra are shown in Fig. 1. Note the substantial, order-unity, difference between the exact and approximate (Limber) expressions at $\ell = 1$; in the remainder of this work we use the exact, double-integral expression.

2.3 2D-projected dipole: kinematic dipole

A dipole pattern may also arise due to motion of the Earth with respect to the astrophysical objects or structure being measured. This is what produces the dipole in the CMB, and it also contributes to the total dipole in a measurement of the LSS.

2.3.1 Kinematic dipole in the CMB

Probably the best known dipole in all of cosmology is the dipole measured in the CMB temperature distribution. This dipole, which has an amplitude of the order of 10^{-3} times the amplitude of the CMB monopole, arises due to the motion of the Solar system with respect to the CMB rest frame. This motion is the vector sum of several different motions, summarized in Table 1.

Values of the kinematic dipole in the CMB are cited with the contribution from the Earth's motion around the Sun subtracted out, so that the dipole is due only to the Sun's velocity with respect to the CMB (Kogut et al. 1993). The value of the Local Group's peculiar velocity with respect to the CMB is from Maller et al. (2003) and was computed using the value of the Sun's velocity with respect to the Local Group in Courteau & van den Bergh (1999). [When the velocity of the Local Group with respect to the CMB rest frame is inferred from the measurement of the CMB dipole, the direction becomes $(l, b) = (276, 30) \pm (3, 3)$ (Kogut et al. 1993). Compare also values determined in Tully et al. (2008) and Bilicki et al. (2011).] The peculiar velocity predicted from linear-theory Λ CDM is $\sim 470 \text{ km s}^{-1}$ (Bilicki 2012). Note that the speed of the Sun with respect to the CMB rest frame would be considerably greater if not for the fact that the Sun's velocity vector with respect to the Local Group points in a direction nearly opposite to that of the Local Group's velocity vector with respect to the CMB. Also note that the dominant contribution to the Sun's motion with respect to the Local Group is the Sun's motion around the centre of the Galaxy, which has speed $\sim 220 \text{ km s}^{-1}$, and is composed of the Sun's motion with respect to the local standard of rest (LSR) and the LSR's motion with respect to the Galactic Centre (Courteau & van den Bergh 1999; Itoh et al. 2010).

2.3.2 Kinematic dipole in LSS

The kinematic dipole in the CMB, which is due to the Sun's motion with respect to the CMB rest frame, is observed as a Doppler shifting of the CMB photons. The effect that gives rise to a kinematic dipole in the LSS is not quite as direct. Rather, it includes contributions both from the Doppler effect and relativistic aberration. We derive the relevant equations in Appendix B.

2.4 2D-projected dipole: intrinsic dipole

In the CMB, the intrinsic dipole corresponding to adiabatic perturbations is zero (Erickcek et al. 2009). When we switch over from talking about the CMB to talking about LSS, we expect that there may be an intrinsic dipole in the LSS. Below, we explore possible reasons why an intrinsic dipole might compete with or even (conceivably) dominate the LSS kinematic dipole.

Erickcek et al. (2009) propose a scenario in which the curvaton (particle mediating a scalar field that may generate fluctuations during inflation without actually driving inflation) has a large-scale spatial gradient, which in turn causes variation in the amplitude of

Table 1. Motions that give rise to a kinematic dipole in the CMB and large-scale structure.

Motion	Approximate speed (km s^{-1})	Direction
Earth around Sun	~ 30	Annually varying
Sun w.r.t. Local Group	~ 306	$(l, b) = (99, -4) \pm (5, 4)$
Local Group w.r.t. CMB	~ 622	$(l, b) = (272, 28)$
Overall CMB kinematic dipole	~ 370	$(l, b) = (264.4, 48.4) \pm (0.3, 0.5)$

the primordial curvature perturbations, modulating Δ_R across the sky. Hirata (2009) shows how this modulation due to isocurvature perturbations would transfer to the CMB and LSS, in the latter case causing a dipolar variation in the abundance of massive haloes (and objects that occupy them). This inflationary scenario is one scenario that invokes the physics of the early universe to explain why there might be an intrinsic dipole in the LSS above and beyond what we naturally expect to be present from typical scale-invariant fluctuations/adiabatic perturbations laid down in the simplest inflationary scenarios. While the simplest curvaton-gradient model has been ruled out by Hirata's analysis of constraints on the dipole in Sloan Digital Sky Survey (SDSS) quasars, and corresponding constraints on dipolar modulation of the primordial power spectrum,³ similar but more complicated scenarios are still possible.

Note that Hirata's constraints on the primordial dipole amplitude using SDSS quasars are of the order of 2×10^{-2} , which corresponds to constraints on the amplitude of the dipole in the quasars themselves roughly an order of magnitude higher. Hence current constraints on this particular intrinsic-dipole scenario are not down to the level associated with the kinematic dipole, though this was not a problem in Hirata's analysis since that analysis looked specifically for a dipole effect that would accompany the primordial conditions needed to explain the CMB hemispherical power anomaly given the curvaton-gradient model, and that would have required a 10^{-1} dipole.

Another possibility for generating an intrinsic dipole is that the CMB rest frame is not the same as the LSS rest frame. This happens, for example, in models with a large (\sim Gpc radius) underdense void, in which we are located close to the centre. The observed bulk flow is then equal to the difference between the Hubble parameters inside and outside the void multiplied by our distance away from the centre⁴ (Tomita 2000). Requiring that the intrinsic dipole thus measured is consistent with observations of the CMB dipole [$v/c \simeq O(10^{-3})$] requires that we live very close ($\lesssim 15$ Mpc) to the centre of the void (Alnes & Amarzguioui 2006), making these models rather fine-tuned.

An alternate mechanism for how a CMB–LSS rest-frame disagreement may arise is provided by the Grishchuk–Zel'dovich effect (Grishchuk & Zeldovich 1978; Erickcek, Carroll & Kamionkowski 2008a; see also Gunn 1988). As Turner (1991) explains, if inflation lasts only a little longer than necessary to solve the flatness and horizon problems, scales that were superhorizon sized at the onset of inflation – these scales cannot be affected by events during or later than the inflationary epoch, and thus contain imprints of the pre-inflationary universe – may not be much larger than our present horizon, and thus may have some effect in the current universe. In particular, he proposes that large density fluctuations with wavelengths slightly larger than the Hubble radius (modes that are 'just barely' superhorizon sized) may exist, and would appear to us as a density gradient in a particular direction. Such a density gradient could produce a 'tilted universe': a universe in which all the matter within the Hubble volume gains a peculiar velocity due to the greater

gravitational attraction from one 'side' of the universe than the opposite. The effect is equivalent to saying that the rest frame of the CMB is not the same as the rest frame of LSS: from the rest frame of the CMB, all matter would have a non-zero average streaming velocity. This would produce an intrinsic dipole due to relativistic aberration and the Doppler effect (or, equivalently, it would produce an additional kinematic dipole on top of that expected from analysis of the CMB) (Itoh et al. 2010). In order for this 'tilting' effect to be observable, isocurvature modes must be present, even in the presence of late-time acceleration (Zibin & Scott 2008).

Given the presence of isocurvature modes, the Grishchuk–Zel'dovich effect would also produce an (additional) intrinsic dipole due to the simple fact of the superhorizon-scale density perturbation. As another example of physical mechanisms that would produce an intrinsic dipole, Gordon et al. (2005) examine a scenario in which there are spatial perturbations in the density of dark energy from a quintessence field: that is, a late-time effect produces horizon-scale fluctuations. More generally, Gordon et al. examine a class of models in which the full fundamental theory is homogeneous and statistically isotropic, but statistical isotropy is broken from a given observer's position because of superhorizon-scale perturbations that appear as a gradient in density across the sky on the largest observable scales. Any theory that generates such a variation in density would give rise to what we have termed an intrinsic LSS dipole, and the appearance of the breaking of statistical isotropy. These density variations could, at least theoretically, exist on essentially any order of magnitude in $\delta\rho/\rho$.

There is some reason to take the idea of a tilted universe seriously. Kashlinsky et al. (2008) investigate the bulk motion of galaxies in the universe out to ~ 300 Mpc h^{-1} and find, somewhat controversially (see e.g. Keisler 2009; Osborne et al. 2011; Mody & Hajian 2012), that there is a coherent bulk flow in their sample. The evidence they develop for this claim comes from attempting to detect the kinetic Sunyaev–Zel'dovich (SZ) effect by computing the dipole of the CMB temperature field evaluated at the positions of galaxy clusters. This dipole, evaluated as it is in a small number of pixels, does not receive appreciable contributions from our own motion (i.e. from the CMB kinematic dipole due to the Sun's motion with respect to the CMB rest frame), but does receive contributions from instrument noise, the thermal SZ effect, the *intrinsic* CMB dipole, and foreground components. However, contributions other than the kinematic SZ effect are, they argue, accounted for in their analysis, with the thermal SZ effect in particular cancelling out/integrating down when averaged over a large number of clusters. Their conclusion is that the dipole in CMB temperature evaluated at cluster positions is due to the kinetic SZ effect due to the bulk flow of the cluster sample. If this effect is authentic, then it fits well with the tilted-universe scenario: the bulk motion is detectable in LSS but does not generate a primordial dipole CMB component.

Other studies that use measurements of peculiar velocities of local neighbourhood galaxies – typically determined by combining the measurements of their distances and redshifts – find larger-than-expected bulk flows (Watkins, Feldman & Hudson 2009; Feldman, Watkins & Hudson 2010). These flows are of the order of 400 km s^{-1} and seem to be showing no signs of convergence out to the probed distance $R \sim 60 h$ Mpc $^{-1}$; they are estimated to be ~ 1 per cent likely in the standard Λ CDM cosmological model for the given observed volume. The relation of these larger-than-expected bulk-flow measurements to findings by Kashlinsky et al. (2008) is unclear at this time (Feldman et al. 2010), especially given that bulk flows inferred from distances obtained from Type Ia supernovae indicate somewhat lower bulk flows that are therefore in better agreement

³ According to Hirata, any smooth gradient in the amplitude of the primordial curvature perturbations is no more than 2.7 per cent per present-day horizon radius (99 per cent confidence); cf. the 11 per cent variation required in the Erickcek et al. model needed for consistency with the CMB hemispherical power anomaly.

⁴ Therefore, the CMB frame and the LSS frame *inside the void* (but not the global LSS frame) are different in this case. Most measurements of bulk flows, being much shallower than the size of the void, would interpret this as a legitimate difference between the CMB and LSS frames.

Table 2. Reviewing the sources of dipole signal in a large-scale structure survey.

	Local-structure dipole	Kinematic dipole	Intrinsic dipole
Typical size	~ 0.1 to 10^{-5} depending on z_{\max}	$\sim 10^{-3}$	$\gtrsim 10^{-5}$
Flux-weighted method probes	Local Group's acceleration; Section 2.1	N/A	N/A
2D-projected method probes	Small-scale departures from statistical isotropy; Section 2.2	Our motion; Section 2.3	Many possible theoretical explanations; Section 2.4

with Λ CDM (Colin et al. 2011; Dai, Kinney & Stojkovic 2011; Turnbull et al. 2012).

2.5 Types of dipoles: review

In summary, when we observe some tracer of LSS [galaxies, quasars, gamma-ray bursts (GRBs), etc.], we may observe a dipole in counts. If the dipole we are observing is what we have called the 2D-projected dipole – that is, a dipole in surface density of the object – then contributions to that dipole may come from (1) the local-structure dipole, (2) the kinematic dipole (which is due to the Doppler effect and relativistic aberration) and (3) an intrinsic LSS dipole (which is really the $z \gg 1$ limit of the local-structure dipole). There are a couple observations to make about these effects.

(i) In the limit of very high redshift, the local-structure dipole goes to amplitudes of the order of 10^{-5} (as we have explicitly verified), the kinematic dipole is expected to go to amplitude $\sim v/c \sim 10^{-3}$ and align with the direction of the CMB dipole, and the intrinsic dipole could take on a wide variety of values depending on certain theoretical considerations.

(ii) For structures/galaxies at relatively small redshifts, the local-structure dipole amplitude much greater than the kinematic dipole amplitude. However, even though the kinematic dipole is swamped by the local-structure dipole, we expect that these two dipoles should point in somewhere close to the same direction. While no particular level of agreement is guaranteed, the fact remains that local structure is what accelerates us in the direction that the kinematic dipole points. This is why, in linear theory, the velocity of the Local Group is proportional to its acceleration due to gravity. However, since the 2D-projected dipole takes no radial information into account, it is not a true measure of gravitational attraction or acceleration, but only a partially reliable proxy.

In Table 2, we show the types of dipoles considered in this work, and the section in which they are introduced.

3 FORMALISM FOR DETECTING DIPOLES

Some forays have already been made into tests of statistical isotropy, and dipoles in particular, using measurements of LSS. Many estimators for the dipole have been employed, some of which do better jobs than others at naturally incorporating sky cuts, allowing for systematic effects to be accounted for, etc. Here we adopt the estimator used by Hirata (2009) to test the *WMAP* hemispherical power anomaly using quasars detected by the SDSS. This is the best unbiased estimator for determining the amplitude and direction of a dipole in counts of objects on the sky under conditions of cut skies and in the presence of systematics. We now describe this formalism.

3.1 Obtaining the direction of the dipole

Consider a dipolar modulation on the sky with some amplitude A in a (unit) direction \hat{d} . We may write the observed density field N of the objects in question as a function of direction \hat{n} as

$$N(\hat{n}) = [1 + A(\hat{d} \cdot \hat{n})]\bar{N} + \epsilon(\hat{n}), \quad (10)$$

where \bar{N} is the intrinsic statistically isotropic field and ϵ combines random and instrumental noise. If we momentarily drop the ϵ term, we can write $\delta N / \bar{N} = A(\hat{d} \cdot \hat{n})$. Reinstating a term corresponding to systematic errors, the fluctuations in density as a function of direction can be written as the sum of contributions from a dipole, fluctuations due to systematics and a mean offset (Hirata 2009):

$$\frac{\delta N}{\bar{N}}(\hat{n}) = A\hat{d} \cdot \hat{n} + \sum_i k_i t_i(\hat{n}) + C. \quad (11)$$

Here $t_i(\hat{n})$ are possible systematics templates in the sky map (such as an extinction map), the coefficients k_i give the amplitudes of the contributions of these systematics to the observed density field, and the presence of the monopole term, C , allows us to account for covariance between the monopole and other estimated parameters, especially covariance between the monopole and any systematic templates.

It is then straightforward to write down the best linear unbiased estimator of the combination (\hat{d}, k_i, C) with corresponding errors. The procedure is as follows. First, we rewrite the above equation as

$$\frac{\delta N}{\bar{N}}(\hat{n}) = \mathbf{x} \cdot \mathbf{T}(\hat{n}), \quad (12)$$

where $\mathbf{x} = (d_x, d_y, d_z, k_1, \dots, k_N, C)$, $\mathbf{T}(\hat{n}) = (n_x, n_y, n_z, t_1(\hat{n}), \dots, t_N(\hat{n}), 1)$ and $n_x^2 + n_y^2 + n_z^2 = 1$.

The best linear unbiased estimator of \mathbf{x} is

$$\hat{\mathbf{x}} = \mathbf{F}^{-1} \mathbf{g}, \quad (13)$$

where the components of the vector \mathbf{g} are

$$g_i = \int T_i(\hat{n}) \delta N^\Omega(\hat{n}) d^2 \hat{n} \quad (14)$$

and the Fisher matrix \mathbf{F} is given by

$$F_{ij} = \bar{N}^\Omega \int T_i(\hat{n}) T_j(\hat{n}) d^2 \hat{n}, \quad (15)$$

where $N^\Omega \equiv dN/d\Omega$ is the number of galaxies per steradian. To actually compute these quantities with discretized data, it is convenient to work with a data map and a random map, the latter of which is simply a set of randomly chosen directions/points \hat{n}_R on the unit sphere:

$$g_i = \sum_D T_i(\hat{n}_D) - \frac{N_D}{N_R} \sum_R T_i(\hat{n}_R), \quad (16)$$

$$F_{ij} = \frac{N_D}{N_R} \sum_R T_i(\hat{n}_R) T_j(\hat{n}_R), \quad (17)$$

where N_D and N_R represent galaxy counts rather than the number of galaxies per steradian as in the continuous case.

Note that the component of \mathbf{g} corresponding to the monopole term in equation (11), which we will refer to as \mathbf{g}_C , must be zero, even if the sky is cut. This can be seen in the analytic formula for \mathbf{g} by noting that we are integrating fluctuations relative to the mean, where the mean is determined from whatever portion of the sky is being integrated over. In the formulation where we discretize the celestial sphere, $\mathbf{g}_C = N_D - (N_D/N_R)(N_R) = 0$; \mathbf{g}_C represents the monopole of the fluctuations from the mean on the cut sky, which must be zero. Hence, the only way the monopole term C in equation (11) can be non-zero is by picking up on the covariances between variables.

To show explicitly how we calculate the Fisher matrix F_{ij} in the discrete formalism, we take F_{zz} as an example:

$$F_{zz} = \frac{N_D}{N_R} \sum_{i=1}^{N_R} z_i^2 = \left(\frac{N_D}{N_R} \right) (N_R) \langle z^2 \rangle = N_D \langle z^2 \rangle, \quad (18)$$

where we have used z to designate the z -coordinate of the vector pointing to the centre of the pixel in which count i is found. Since $\langle z^2 \rangle = 1/3$ over the entire sphere, we have $F_{zz} = N_D/3$ (for the entire celestial sphere) in the limit of sufficiently large number of counts in the random map to have suppressed Poisson noise.

3.2 Errors in estimated dipole direction

The matrix F_{ij} is the Fisher matrix for the full parameter set $p_i = \{d_x, d_y, d_z, k_1, \dots, k_N, C\}$, and hence the covariance matrix is $\text{Cov}(p_i, p_j) = (F^{-1})_{ij}$. By the Cramér–Rao inequality, the best case marginalized errors on the parameters are

$$\sigma_{\text{marg}}(p_i) = \sqrt{(F^{-1})_{ii}}, \quad (19)$$

inverting F automatically mixes all the elements together and takes into account how they covary. Meanwhile, the best case unmarginalized errors are

$$\sigma_{\text{unmarg}}(p_i) = 1/\sqrt{F_{ii}}. \quad (20)$$

Note that the errors on our estimates of the dipole are based on the shape of the sky cut, the input systematic templates and the number of data points N_D .

As a side note, the correlation between parameters p_i and p_j is

$$\rho_{ij} = \frac{F_{ij}^{-1}}{\sqrt{(F_{ii}^{-1} F_{jj}^{-1})}}. \quad (21)$$

3.3 Estimating the amplitude of the dipole

In this formalism, we need only to acquire the components of the dipole (d_x, d_y, d_z), and the associated errors ($\sigma_x, \sigma_y, \sigma_z$). Combining the components by squaring, summing and taking the square root of the sum would create a biased estimator of the dipole amplitude A , so we never do this. Instead, once we have the best-fitting dipole $\mathbf{d}_{\text{best}} \equiv A\hat{\mathbf{d}}$, we can construct a marginalized likelihood function for the amplitude A (Hirata 2009):

$$\mathcal{L}(A) \propto \int \exp \left[-\frac{1}{2} (A\hat{\mathbf{n}} - \mathbf{d}_{\text{best}}) \text{Cov}^{-1} (A\hat{\mathbf{n}} - \mathbf{d}_{\text{best}}) \right] d^2\hat{\mathbf{n}}, \quad (22)$$

where $d^2\hat{\mathbf{n}}$ indicates integration over all possible directions on the sphere.⁵ In this equation, for a given amplitude A , we take the best-fitting dipole \mathbf{d}_{best} as given, and then compare each direction $\hat{\mathbf{n}}$ (with the given amplitude) with the best-fitting dipole. The likelihood function is a Gaussian in A for fixed direction $\hat{\mathbf{n}}$, by construction, but may not be Gaussian when marginalized over direction. That marginalization occurs in the equation above when we integrate over all $\hat{\mathbf{n}}$, giving us the likelihood of a particular amplitude A marginalized over all directions, given the best-fitting dipole \mathbf{d}_{best} ‘selected’ by the data. Posterior analysis will then show where 95 per cent of the weight lies.

Given that we ultimately work discretely, with a celestial sphere that is pixellized using HEALPIX (Górski et al. 2005), the likelihood turns into a sum over pixels:

$$\mathcal{L}(A) \propto \sum \exp \left[-\frac{1}{2} (A\hat{\mathbf{n}} - \mathbf{d}_{\text{best}}) \text{Cov}^{-1} (A\hat{\mathbf{n}} - \mathbf{d}_{\text{best}}) \right] \Delta \text{Area} \quad (23)$$

The factor ΔArea will come out of the summation since all pixel areas are equal in HEALPIX and all that matters is the ratio of likelihoods rather than the absolute values of likelihoods, so we literally sum over all the pixels in order to get the marginalized $\mathcal{L}(A)$. We drop the prefactor on the likelihood that includes covariance since the covariance does not depend on parameters.

Finding the likelihood distribution as a function of direction, $\mathcal{L}(\hat{\mathbf{n}})$, follows from an exactly analogous procedure, but we sum over all possible amplitudes associated with a given pixel rather than over all possible pixels associated with a given amplitude.

3.4 Converting from dipole amplitude A to angular power spectrum C_1

Here we show that there is a simple relationship between the dipole power C_1 , the $\ell = 1$ mode of the angular power spectrum familiar from several areas of cosmology, and the amplitude A of the dipole computed above. Without loss of generality, we assume that the dipole points in the positive z direction and write the fluctuation in counts in two ways:

$$\frac{\delta N}{N}(\hat{\mathbf{n}}) = A\hat{\mathbf{n}} \cdot \hat{\mathbf{d}} = A \cos \theta = a_{10} Y_{10}(\hat{\mathbf{n}}), \quad (24)$$

where $Y_{10} = \sqrt{3/(4\pi)} \cos \theta$. Therefore, $a_{10} = \sqrt{4\pi/3} A$. Using the usual relationship $C_\ell = \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2 / (2\ell+1)$, the power spectrum C_1 contribution is then given by

$$C_1 = \frac{4\pi}{9} A^2. \quad (25)$$

For the purposes of order-of-magnitude calculations, the rule of thumb is $C_1 \simeq A^2$.

3.5 Commentary on the formalism

It is straightforward to show that this estimator is either precisely or approximately equivalent to similar estimators used by recent authors. Nevertheless the formalism of Hirata that we presently use has several practical advantages. First, the real-space estimator

⁵ Even though \mathbf{d}_{best} itself is a biased estimator of the amplitude A , our likelihood as written in equation (22) returns an unbiased estimate. The reason is that the likelihood effectively compares theory to measurement in each component (x, y or z) separately, as can be seen by diagonalizing and rewriting the exponent in that expression, and the components themselves are unbiased. Numerical evidence that the estimate of A is entirely unbiased is shown in Fig. B1, in Appendix B.

employed here is more convenient to implement than multipole-space estimators employed in previous analyses (e.g. Baleisis et al. 1998; Blake & Wall 2002; Frith, Outram & Shanks 2005b). Many analyses use pseudo- C_ℓ to deal with sky cuts, while sky cuts are very straightforward to deal with in this formalism (see Section 3.5 for further details). Finally, estimating the coefficients k_i allows one to very naturally incorporate any systematics templates one suspects might be relevant and ensure that they do not interfere with estimation of the dipole. This form of component separation allows one to isolate the different contributions to the observed fluctuations in counts, and separate those contributions into actual dipole plus systematic effects. Any pattern put into this formalism as a systematic template will be marginalized over in the determination of dipole amplitude A and direction $\hat{\mathbf{d}}$.

Moreover, this formalism has several other noteworthy features as discussed in the following.

(i) It allows very naturally for arbitrary sky cuts: all that is necessary is to remove pixels from both the data map and the random map when performing the dipole analysis. When the sky is cut, the dipole becomes coupled to other multipoles, and the errors on the detection of d_x , d_y and d_z derived from the Fisher matrix correspondingly increase to account for this.

(ii) It allows for straightforward incorporation of arbitrary pixellization. The scale of the pixellization should not matter because Poisson noise is on the scale of the pixellization, which is much smaller than the scale of the dipole, and Poisson noise in larger pixels means a smaller effect (goes as $1/\sqrt{N}$), so the Poisson noise cancels out. However, different pixellization schemes can affect the size, shape and nature of a mask if there is any sky cut, and when discrepancies appear between dipole results using different pixellization schemes, it is virtually always traceable to this.

(iii) It allows for the possibility of $A > 1$. This may seem counterintuitive since it implies negative counts in some pixels. However, even though it is true that counts cannot go negative in real data, it is still possible for a *model* in which some pixels have negative counts to be the best fit to the data.

Note that the level of degeneracy between the systematic template $t_i(\hat{\mathbf{n}})$ and the dipole $\hat{\mathbf{d}}$ depends on structure of the former, and on the relative orientation of the two. We have performed extensive tests to verify and build our intuition about this degeneracy. For example, if a pure-dipole map is created with the dipole pointing in the z -direction, and this map is used as a template $t(\hat{\mathbf{n}})$, the effect is that the detection of the dipole in the z -direction, d_z , gives an unreliable number, while the error bar in the z -direction, σ_z , blows up to a much larger number than d_z , so that $(S/N)_z = d_z/\sigma_z \ll 1$. Also, the correlation ρ_{zk} between d_z and the template coefficient k becomes 1.0. In other words, a dipole template in a given direction takes out any component of the dipole detection in that direction.

Finally, we performed tests with varying sky coverage and survey depth, and the number of (mock) galaxies available to verify that the input dipole is successfully recovered within the estimator's reported error.

We proceed to apply this estimator to data from several surveys. We select surveys with very wide sky coverage, ideally almost full-sky coverage, because for a fixed depth, the number of modes available scales as the fraction of the sky covered, f_{sky} . This is useful especially for beating down cosmic variance in theoretical predictions, which as we will see is the dominant source of uncertainty in our comparisons of observations with theory (observational results tend to be much more tightly constrained than theoretical predictions since we are working at very low ℓ).

We also find that higher multipoles do not contribute appreciably to the recovered signal, or even strongly affect the error bars on the dipole signal, as long as more than roughly half the sky is probed in the survey. See Appendix B for details of how we use maps of $\ell = 2$ and 3 modes as systematics templates to detect the presence of coupling among multipoles.

Previous research on dipoles in similar surveys will be profiled as different types of surveys are brought up.

4 DIPOLE IN 2MRS

4.1 Introduction to dipole signals in 2MASS

2MASS, which imaged 99.998 per cent of the celestial sphere (Skrutskie et al. 2006), provides an excellent starting point in testing for dipoles in tracers of LSS. This survey includes two main catalogues, the point-source catalogue (PSC) and extended-source catalogue (XSC). The latter is of interest here since it includes roughly 1.6 million sources, nearly all of which are extragalactic.

2MASS used two 1.3-m equatorial Cassegrain telescopes, one in the Northern hemisphere and one in the Southern hemisphere (Mt Hopkins, Arizona; Cerro Tololo, Chile), to observe in the J , H and K_s bands, corresponding to wavelengths of 1.25, 1.65 and 2.16 μm , respectively. The XSC contains sources that are extended with respect to the instantaneous point spread function (Skrutskie et al. 2006), including galaxies and Galactic nebulae. The $S/N = 10$ sensitivity limits are met by sources as bright or brighter than 15.0, 14.3 and 13.5 mag in the J , H and K_s bands, respectively, and (very importantly for our dipole-related considerations) exhibit a mean colour difference of less than 0.01 mag between hemispheres, meaning that the photometry is highly uniform between hemispheres. The reliability (corresponding to the ratio of the number of genuine extended sources to the total number of sources, spurious or genuinely extended, in the data set) of the XSC is greater than 99 per cent for Galactic latitude $|b| > 20^\circ$. Some extended sources in the catalogue are not extragalactic, though these can be easily removed with the right colour cuts (as detailed later in Section 5).

A small subset of the 1.6 million extended sources in the 2MASS XSC were assigned redshifts in the 2MRS, a catalogue which includes position and redshift information for over 40 000 galaxies present in the original 2MASS sample. In this section and the next, we apply the dipole-detecting formalism outlined in Section 3 to the entire 2MRS catalogue, as well as to appropriate subsets of the 2MASS XSC sources.

Erdogdu et al. (2006), Maller et al. (2003) and Bilicki et al. (2011), for example, have calculated the flux-weighted dipoles (see Section 2.1) for 2MRS (a 23 000 galaxy subset thereof, actually, with $K_s < 11.25$ – a preliminary version of the catalogue) and 2MASS, respectively. This stands in a longer tradition of attempting to calculate flux-weighted dipoles from near-infrared surveys since near-infrared light closely traces the mass distribution of LSS. For instance, Rowan-Robinson et al. (2000) calculate a flux-weighted dipole from the *IRAS* PSCz Redshift Survey, which had redshifts for over 15 000 *IRAS* galaxies (at 60 μm ; cf. the wavelengths of 2MASS, over an order of magnitude shorter). The *IRAS* PSCz ($z_{\text{max}} \sim 0.1$), 2MRS ($z_{\text{max}} \sim 0.1$) and 2MASS XSC ($\bar{z} > 0.07$) studies all find tolerably small discrepancies between the direction of the flux-weighted dipole (and thus the acceleration of the Local Group) and the CMB velocity dipole that partially results from that acceleration (velocity of the Local Group being proportional to acceleration of the Local Group in linear theory). (As noted before, the motion of the Sun with respect to the Local Group also contributes to the

kinematic dipole, but the direction is nearly opposite the direction of the Local Group's motion with respect to the CMB rest frame, and hence changes the magnitude of the velocity vector but does not substantially change its direction.)

The number-weighted dipole of Erdogdu et al. (2006) comes closer than the flux-weighted dipole to mimicking the quantity that we calculate here, but the number-weighted dipole, like the closely related flux-weighted dipole, is another quantity that seeks to measure the acceleration of the Local Group due to surrounding LSS, but instead of using flux as a proxy for mass as in the flux-weighted dipole, the number density of galaxies in a given direction on the sky is assumed to serve as a good proxy for mass. Our goals and aims, as well as the precise quantity we calculate, are different: we seek to measure not an acceleration, but rather the simple 2D-projected dipole, which, in the case of the relatively nearby survey 2MASS (and 2MRS, as a subset of 2MASS), is dominated completely by the contributions from the local-structure dipole (see Section 2.2).

The entire power spectrum of 2MASS has been calculated by Frith et al. (2005b), which means that at least one measure of the 2D-projected dipole that we explore here has already been obtained. We compare our results to this previous result later in this section. However, we do not regard our result as a simple replication of the previous result (and, in fact, we note substantial disagreement): we compute not just the amplitude of the dipole, but also its direction; we account for systematics in a direct and natural way; and we place the 2MASS dipole into a larger context of exploring the various contributions to dipoles, and testing observational results against theoretical predictions, in a wide variety of surveys.

4.2 2MRS profiled

We begin with the 2MASS Redshift Survey, the densest all-sky redshift survey to date. The 2MRS team (Huchra et al. 2011) measured redshifts of 43 533 bright ($K_s < 11.75$) sources with $E(B - V) \leq 1$ mag and $|b| \geq 5^\circ$ (for $30^\circ \leq l \leq 330^\circ$; $|b| \geq 8^\circ$ otherwise). Sources were carefully screened to ensure that all were genuinely extragalactic sources and do not have compromised photometry. As explained below, we err on the conservative side and make a symmetric cut at $|b| < 8^\circ$ for all Galactic longitudes l , which eliminates roughly 1700 of the galaxies in the survey. Nearly all 2MRS galaxies are within the range $0 < z < 0.1$. Previous tests have worked with the flux-weighted dipole in 2MRS (Erdogdu et al. 2006), but have not explored the various contributions to the 2D-projected dipole.

The results for the dipole amplitude in this survey are strongly expected to agree with theoretical predictions. Given the relatively small volumes surveyed (and the fact that we are dealing with the very large scale $\ell = 1$ mode), cosmic variance is very large, and so a statistically significant discrepancy between theory and observation would require highly anomalous disagreement between the two. We find, even with relatively cursory checks, that there are no serious discrepancies between theory and observation for 2MRS. Nevertheless, in the next sections, we profile the various tests performed on the data, and the results for both dipole direction and amplitude, in the interests of presenting results for this survey as something of a model (in addition to being an important test in its own right): this is a data set with well-controlled systematics and very little chance of giving anomalous results, where we can test out several different types of systematic checks, to gain intuition for what results should look like when we perform similar tests on surveys at higher redshifts and/or with less well understood systematics.

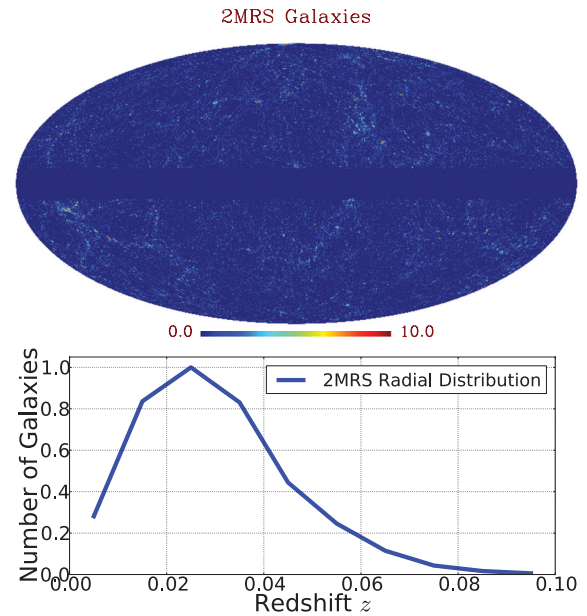


Figure 2. Top panel: all sources in the 2MASS Redshift Survey that escape the $|b| < 8^\circ$ cut. The mean redshift in the survey is approximately $\bar{z} = 0.028$. Even by eye, it is clear that the dipole due to local structure has not died away at these scales. In particular, the supergalactic plane is still fairly clearly visible in the data (see e.g. Maller et al. 2003). (Note that the dynamic range of this plot has been limited so that structures outside the supergalactic plane are also visible.) Bottom panel: a plot of the radial distribution of 2MRS galaxies. The data are put in redshift bins $[0, 0.01)$, $[0.01, 0.02)$, \dots , $[0.09, 0.10)$, where the plot shows the number of galaxies in each bin as a function of bin centre.

This survey is essentially ready to be analysed ‘straight out of the box’, meaning that major systematic errors have already been addressed (especially Galactic extinction), and we already have a sample of extragalactic sources with uniform sky coverage outside the Galactic plane. (The latter is important since a lack of uniform completeness across the sky could, if not properly accounted for, mimic the effect of a dipole.) More careful attention must be paid to these matters in the 2MASS sample as a whole, and in other surveys, but 2MRS requires only that we cut out pixels (in both the data map and the random map to which it is compared; see Section 3) within 8° of the Galactic equator, $|b| < 8^\circ$. Note that pixels whose centres are above 8° , and thus escape a straightforward cut of pixels with centres below 8° , still may have area below 8° , especially if the pixelization is coarse, as in the cases of HEALPIX resolution⁶ NSIDE = 8 or 16. We adopt NSIDE = 128 for the rest of this paper, except where otherwise noted, and also cut conservatively so that pixels with any area at all with $|b| < 8^\circ$ are cut.

4.3 Observational constraints on dipole amplitude as a function of redshift

With $|b| < 8^\circ$ excised from the map (see Fig. 2), we can apply the formalism outlined in Section 3 directly to different subsets of the survey. We pixelize the data using HEALPIX, meaning that we take the Galactic coordinates given in survey data and assign the given galaxy to the pixel corresponding to those coordinates. Fig. 3 gives

⁶ Resolution at some NSIDE roughly corresponds to pixel size $\theta = 1^\circ \times (60/\text{NSIDE})$.

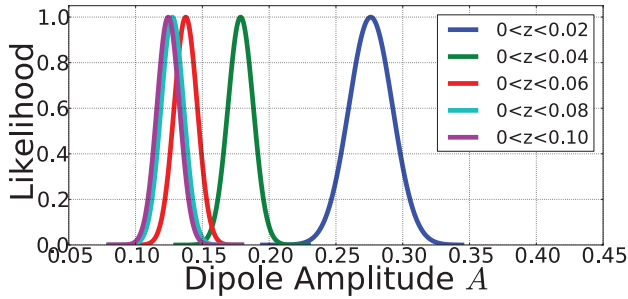


Figure 3. Likelihood curves for different maximum redshifts in 2MRS. Any galaxies with $|b| < 8^\circ$ are removed from the sample.

the likelihood of the dipole amplitude, $\mathcal{L}(A)$, for different subsets of the entire survey, reaching out to $z_{\max} = 0.02, 0.04, 0.06, 0.08$ and 0.10 (where the last value in that list represents the entire survey except for 25 sources at an assortment of higher redshifts).

Note that the behaviour is as expected in several regards as discussed in the following.

- (i) The dipole amplitude A starts off larger and grows smaller as we go out further in redshift.
- (ii) A converges to a certain value. This should happen simply because we run out of sources as we go to higher and higher redshift (e.g. 41 446 sources are at $z < 0.06$, while the total number of sources with $z < 0.10$ is 43 506.)
- (iii) Because the redshifts in this sample are relatively low in cosmological terms, going out only to $z \sim 0.10$, we expect that the dipole amplitude should remain of the order of 10^{-1} , and it does.

Although 2MRS should be relatively systematic-free, we proceed to perform straightforward tests for two types of systematic effects: Galactic extinction as characterized by the maps of Schlegel, Finkbeiner & Davis (1998), and star–galaxy confusion or other systematics that vary as a function of Galactic latitude b .

4.4 Comparison of dipole parameters with and without extinction template

If we wish to take Galactic extinction into account, the formalism we are employing to find dipole amplitudes and directions allows for very straightforward incorporation of extinction due to dust as a systematic template. The 2MRS maps are already extinction-corrected, so this is much more of a sanity check than anything else.

We use the SFD map (Schlegel et al. 1998), an intensity map of the sky at $100 \mu\text{m}$ and a reprocessed composite of the *COBE*/DIRBE and *IRAS*/ISSA maps. The resolution is of the order of a few ar-

cminutes, and so the SFD map can in general be used to reliably derive extinction due to dust, assuming a standard reddening law. This works best away from the plane of the Galaxy since within the Galactic plane dust conditions tend to fluctuate much more strongly on small scales than they do away from the Galactic plane (with the possibility of multiple dust temperature distributions, variable grain sizes, etc.). However, since we work almost exclusively well away from the Galactic equator, we do not expect this to be an issue.

The SFD map is nearly parity-even in Galactic coordinates, as the Galaxy is itself nearly parity-even, so when the sky is symmetrically cut, extinction is not likely to contribute to, or diminish, a dipole-like (parity-odd) signal, at least not in the z -direction. However, we still include it as a precaution against a known potential source of systematic error.

The results are best presented in the form of a direct comparison in Table 3. Entries in this table take into account the entire 2MRS sample, 43 506 galaxies (before the symmetric cut in Galactic latitude) with $0.00 < z < 0.10$.

The results change only slightly when the SFD template is added, and are statistically consistent with the no-dust results. The fact that the dipole amplitude drops slightly with addition of the template is an indication that a very small amount of the dipole power in the 2MRS map can be attributed to the pattern set up by the distribution of dust in our Galaxy.

The final row of Table 3 gives the results on dipole amplitude and direction when we include not only the SFD map but also the five $\ell = 2$ and seven $\ell = 3$ modes as templates. This ensures, as discussed earlier and elaborated upon in Appendix B, that we are detecting only dipole signal and are not receiving contributions from higher multipoles. The contributions from higher multipoles are almost negligible in this case, though they do slightly affect the direction of the detected dipole signal.

4.5 Dipole parameters as a function of sky cut

With the SFD extinction template in place, and again using the entire 2MRS sample out to $z = 0.10$, we may also vary the location where the cut in Galactic latitude is placed. Verifying that the placement of the cut (as long as it is kept at least as aggressive as the $|b| < 8^\circ$ cut) does not affect the results beyond widening our error bars serves as a check for any source of systematic error that varies as a function of Galactic latitude. Most notably, any star–galaxy confusion that might creep into the survey (very unlikely in the case of this particular survey) would vary strongly as a function of Galactic latitude, with the density of stars dropping precipitously as one moves away from the Galactic equator, and so this test serves to verify that star–galaxy confusion is not a major contributor to the detection of a dipole. It also helps to guard against the possibility

Table 3. Comparison of dipole parameters with systematics templates versus without templates, for 2MRS. The first column gives the Galactic latitude b of the cut; the second column identifies any systematic template present; the third gives the HEALPIX NSIDE parameter; the fourth gives the number of sources that were still available after the cut was made; the fifth gives the dipole amplitude with highest likelihood; the sixth and seventh give l and b of the best-fitting dipole; and the eighth and ninth give the 68 and 95 per cent confidence intervals (CIs) on the amplitude of the dipole. The first row gives results when we include no systematics templates; the second when we include only the SFD dust map as a template, and the third when we include SFD and quadrupole and octopole modes as templates.

$ b \geq$	Systematics	NSIDE	N	A_{peak}	l	b	68 per cent CI	95 per cent CI
8.0	None	128	41834	0.124	228.0	38.7	0.116–0.132	0.108–0.141
8.0	SFD dust	128	41834	0.118	222.3	38.3	0.110–0.126	0.102–0.135
8.0	SFD dust + Quad + Oct	128	41834	0.120	213.8	35.2	0.111–0.128	0.103–0.137

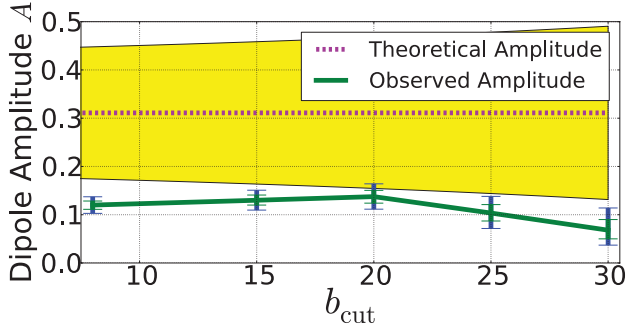


Figure 4. Results for the dipole amplitude as a function of cut in Galactic latitude b . Here b_{cut} indicates that $|b| < b_{\text{cut}}$ was cut out of the map. Notice the very wide cosmic-variance band (yellow shaded region) around the theoretically predicted value for the dipole amplitude, and the consistency of all observed values within cosmic-variance limits. The small green (blue) error bars indicate 68 per cent (95 per cent) measurement errors. The measurement errors are tiny in comparison with cosmic variance. Possible contamination from the quadrupole and octopole, which becomes more important as more of the sky is cut, is taken into account by including all $\ell = 2$ and 3 modes as systematics templates in the analysis.

that variations in sky coverage (see Section 4.10) affect the dipole signal. [Sky coverage is better at higher Galactic latitudes since extended sources cannot be observed very close to very bright stars – less than 2 per cent of the area of a typical high-latitude sky is masked by stars, as noted by Erdogdu et al. (2006).]

Note that as the sky cut becomes more and more aggressive, we expect the error bars on the observed value of A to become wider (simply because we have less data and therefore looser constraints), but we expect the best-fitting/peak-likelihood value of A itself to remain consistent with values found given less aggressive sky cuts. If A shifts in such a way that more aggressive sky cuts yield amplitudes inconsistent with amplitudes from maps with less aggressive sky cuts, this is a fairly good indication that star–galaxy confusion or another systematic effect that varies with Galactic latitude is at play. While the angular dimensions (and thus angular cuts) are not what determine the amplitude of the dipole, for very aggressive cuts we are left with far less data than we are with minimal cuts, and this means that (a) the ‘measurement error’ on the observation becomes greater, and, even more importantly, (b) the cosmic variance associated with the theoretical prediction becomes much greater. Due to (b) in particular, a much wider range of peak-likelihood values for the amplitude becomes consistent with theory, as we cut the sky more aggressively.

Results for different sky cuts are found in Fig. 4. The results are all consistent with one another, and with theory (all measurements being just outside the 1σ cosmic-variance band). Note that we have included quadrupole and octopole modes as systematics templates (see Appendix B) in creating this plot since $\ell = 2$ and 3 modes affect the error bars somewhat for more aggressive cuts.

4.6 Dipole amplitude: theory versus observation

Now that we have established basic consistency among dipole determinations with different sky cuts, we go back to the least restrictive cut, at $|b| < 8^\circ$, and keep the SFD template in place. We proceed to compare theory and observation in dipole amplitude as a function of redshift in 2MRS.

As noted earlier, it is important to take the bias of tracers of the LSS into account when producing theoretical expectations for the

clustering of these objects. Frith et al. (2005b) find the bias in the 2MASS K_s band to be 1.39 ± 0.12 , employing a technique that uses constraints on the galaxy power-spectrum normalization as well as previous constraints on σ_8 . We therefore adopt 1.4 as the value of the bias for both 2MRS and 2MASS in general. The qualitative conclusions drawn from these surveys do not depend strongly on the precise value adopted for the bias. Note that, for constant bias b , dipole amplitude is proportional to the bias, $A \propto b$.

There are cosmic-variance errors on all theoretical predictions:

$$\delta C_\ell = \sqrt{\frac{2}{(2\ell + 1)f_{\text{sky}}}} C_\ell. \quad (26)$$

We can relate C_ℓ to the amplitude A via $C_1 = (4\pi/9)A^2$ and so doing error propagation to get the cosmic-variance error on the amplitude, we have

$$\delta A = \frac{1}{2} \sqrt{\frac{2}{3f_{\text{sky}}}} A. \quad (27)$$

This allows us to plot cosmic-variance uncertainties in both C_1 and A . The basic result of doing so is shown in Fig. 5. Note that the errors on the observations are very small in comparison with cosmic variance. Therefore, for the rest of this section, we consider measurement errors negligible and focus only on how observational results compare with theory within the bounds of cosmic variance.

The dipole amplitude, both theory and measurement, decreases as redshift increases, exactly as it should given our previous arguments that averaging over more structure at larger distances yields lower values of the dipole. Whether the amplitude is expressed as A or C_1 , the observational results are consistently lower than the theoretical expectations. If these measurements for different z_{max} were all independent, there would be a highly significant inconsistency

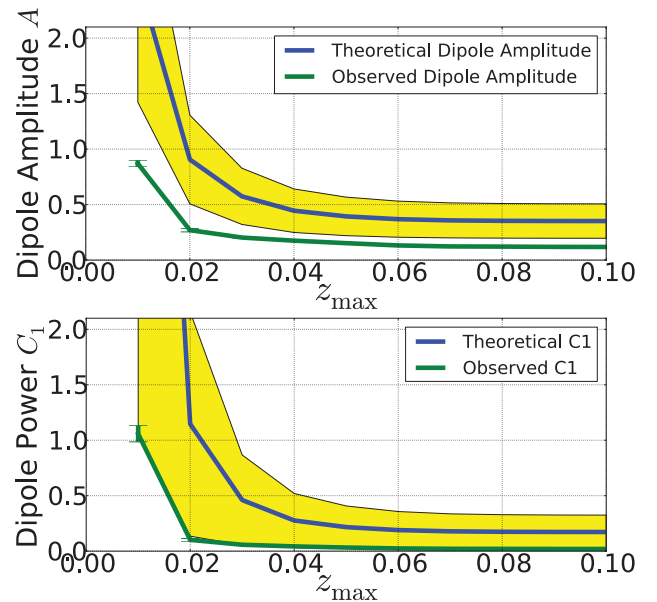


Figure 5. Top panel: comparison of observations with theory for the dipole amplitude, as a function of how much of the 2MRS sample is included ($0.00 < z < z_{\text{max}}$). All observed values are found including the SFD template, and with a cut at $|b| < 8^\circ$. For the purposes of calculating theoretical predictions, we take $f_{\text{sky}} = 0.86$, corresponding to the $|b| < 8^\circ$ cut.) Bottom panel: the same results, only with the dipole power C_1 rather than dipole amplitude A . In all cases, 68 per cent error bars on observations are shown, but are either invisible or nearly invisible due to how tiny they are. See Section 3 for details of the procedure to convert between A and C_1 .

between theory and observation, but the measured values are highly correlated since samples with higher z_{\max} contain all samples with lower z_{\max} .

More specifically, correlations between a bin i going out to $z_{\max,1}$ and another bin j going out to $z_{\max,2}$ are calculated as

$$\text{Cov}[C_{ii}, C_{jj}] = \frac{2}{2l+1} C_{ij}^2, \quad (28)$$

where we have made the usual assumption that the galaxy overdensity is a Gaussian random variable, so that Wick's theorem can be applied to obtain the expression above. Then the correlation coefficient is

$$\begin{aligned} \rho &= \frac{\text{Cov}(C_{ii}, C_{jj})}{\sqrt{\text{Cov}(C_{ii}, C_{ii})\text{Cov}(C_{jj}, C_{jj})}} \\ &= \frac{C_{ij}^2}{(C_{ii}C_{jj})} \end{aligned} \quad (29)$$

Correlations between 2MRS samples range from 0.42 [between the full ($0.00 < z < 0.10$) sample and the smallest ($0.00 < z < 0.01$) sample], to 0.81 [between the full sample and the second-smallest ($0.00 < z < 0.02$) sample], to well over 0.99 for many combinations of samples. (This is also the reason why all bins have similar significance as compared with one another.) Therefore, rather than being a 10σ inconsistency between theory and observation, Fig. 5 represents only slightly more than a 1σ discrepancy. The next section will find the precise ‘discrepancy’ rigorously.

4.7 Comparison of theory and observation for dipole amplitude

Given a Gaussian field on the celestial sphere with observed angular power spectrum C_ℓ^{obs} , the power is χ^2 -distributed, and the likelihood of a given theoretical value C_ℓ^{th} is

$$\ln P(C_\ell^{\text{th}} | C_\ell^{\text{obs}}) = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} \left[-\frac{C_\ell^{\text{obs}}}{C_\ell^{\text{th}}} + \ln \frac{C_\ell^{\text{obs}}}{C_\ell^{\text{th}}} \right] - \ln C_\ell^{\text{obs}} \quad (30)$$

(see e.g. Chu et al. 2005).⁷ Here the observed quantity C_ℓ^{obs} is treated as a realization of the theoretical value C_ℓ^{th} . For $\ell = 1$, this simplifies to

$$\ln P(C_1^{\text{th}} | C_1^{\text{obs}}) = \frac{3}{2} \left[-\frac{C_1^{\text{obs}}}{C_1^{\text{th}}} + \ln \frac{C_1^{\text{obs}}}{C_1^{\text{th}}} \right] - \ln C_1^{\text{obs}}. \quad (31)$$

Again, we treat this as a likelihood, so that P is a function of the theoretical model C_1^{th} , with the observed quantity held fixed. Then, as usual, we can plot the likelihood of a parameter value (in this case, theoretical C_1^{th}) and see where our ‘actual’ theoretical C_1^{th} falls with respect to that distribution. In each redshift bin, we could generate a different likelihood distribution based on the observation, and then compare to the actual C_1^{th} in each case. However, because of the very high correlations between redshift samples, we gain very little by doing tomography in this way, and so we only perform this analysis on the full 2MRS sample, $0.00 < z < 0.10$. The results are shown in Fig. 6.

⁷ This expression can be derived by noting that the random variable $Y = (2\ell+1) \frac{C_\ell^{\text{obs}}}{C_\ell^{\text{th}}}$ is χ^2 -distributed with $2\ell+1$ degrees of freedom. Inserting this expression for Y into the general expression for a χ^2 distribution, and then using the fact that $P(Y)dY = P(C_\ell)dC_\ell$, it is relatively straightforward to show that the proportionality for $P(C_\ell)$ given in Chu et al. (2005) holds, and from there the expression for the log-likelihood given above immediately follows.

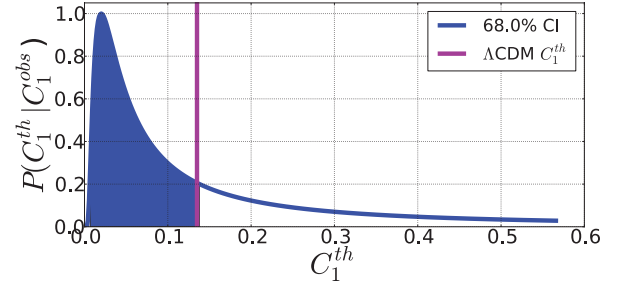


Figure 6. Posterior probability of theoretical C_1 given observed C_1 as a function of the theoretical value, for the full sample of 2MRS galaxies. The observed C_1 determines the likelihood distribution for C_1^{th} , and we can then compare the Λ CDM value (vertical magenta line) for C_1^{th} to that distribution. The Λ CDM value is clearly consistent with the observation.

Whether we calculate a simple signal-to-noise ratio (S/N) to compute the significance of C_1^{th} results given C_1^{obs} , as in Fig. 5, or whether we use the more detailed comparison of Fig. 6, the qualitative conclusion is the same: the Λ CDM prediction matches observations within appropriate cosmic-variance limits. Note that the reason why these two strategies do not match up quantitatively is that the S/N strategy assumes cosmic variance is symmetric, while using the $P(C_1^{\text{th}} | C_1^{\text{obs}})$ distribution takes into account the asymmetry of cosmic variance, particularly at the very low ℓ at which we are working. This is also why significances in C_1 and A do not match up with one another exactly.

4.8 Observational constraints on dipole direction as a function of redshift and sky cut

Up to this point, we have been focusing on the dipole amplitude and comparing theoretical and observed amplitudes. The direction of the dipole, however, is also a quantity of considerable interest.

As discussed in Section 2, there are three major types of dipoles that could contribute to any detected dipole in objects that trace LSS: the local-structure dipole, the kinematic dipole, and the intrinsic dipole. At the scales probed by 2MRS, we expect the local-structure dipole to completely dominate other contributions since it is of the order of 10^{-1} while the kinematic dipole falls two orders of magnitude below this and the intrinsic dipole may very well fall even further below that. Therefore, there is no reason to expect that the direction of the 2MRS dipole should align with the direction of the CMB dipole, as we would expect it to do if the kinematic dipole were dominant at these scales.

That said, it should not be at all surprising if the 2MRS local-structure dipole points somewhere near the CMB kinematic dipole. The reason for this has to do with what generates the motion that gives rise to the kinematic dipole. As discussed in Section 2, the total velocity of the Sun with respect to the Local Group is directed along almost the same line as the velocity of the Local Group with respect to the CMB rest frame, but in the opposite direction. So the direction of the Sun's total motion with respect to the CMB rest frame is essentially the same as the direction of the Local Group's motion with respect to the CMB rest frame, but the speed is lower than that of the Local Group since the contribution of the Sun's motion with respect to the Local Group gets subtracted off. The Local Group moves in a certain direction with respect to the CMB rest frame because of the gravitational pull of structure in the relatively nearby universe. The acceleration due to gravity of the Local Group, as determined via flux-weighted dipole measurements, is directed

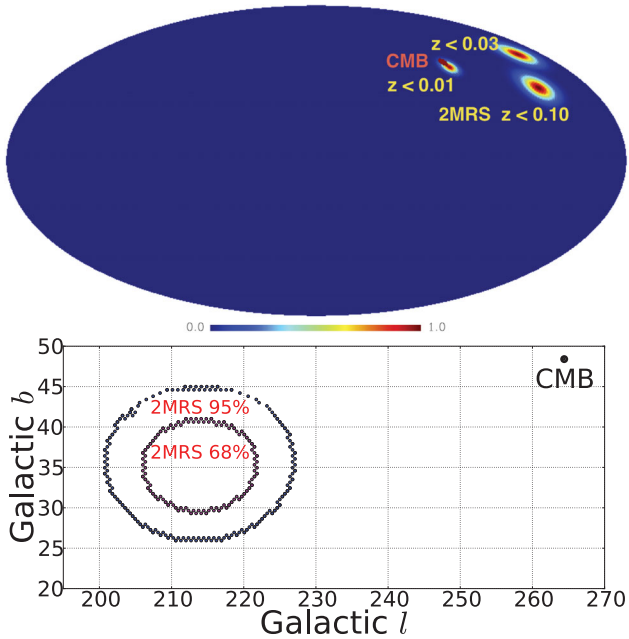


Figure 7. Top panel: likelihood associated with each dipole direction on the sky, marginalized over amplitude, shown for 2MRS redshift shells $0.00 < z < 0.01$ (leftmost multicoloured oval), $0.00 < z < 0.03$ (uppermost multicoloured oval) and $0.00 < z < 0.10$ (rightmost multicoloured oval). We assume a $|b| < 8^\circ$ cut, and incorporate the SFD dust systematic template and quadrupole and octopole templates. The colour scale represents normalized likelihood as a function of direction. The single-coloured disc that overlaps with one of the multicoloured likelihood ovals represents the direction of the CMB kinematic dipole, with error bars exaggerated to a circle of 2° in order to make the position clearly visible on the map. Bottom panel: confidence intervals for the direction of the dipole in the full 2MRS survey, with the position of the CMB dipole shown. Agreement was not expected, but it is reassuring that the 2MRS projected dipole does lie in the same general region of sky as the CMB dipole.

less than 20° away on the sky from the direction of the velocity of the Local Group (Maller et al. 2003). Therefore, insofar as the local-structure dipole gives information about the clustering of local structure and the direction of the acceleration due to gravity of the Local Group, it is expected that it should point in at least the same general direction as the CMB kinematic dipole, which is generated in part by the velocity of the Local Group induced by its acceleration due to gravity. Since the local-structure dipole is a 2D-projected quantity rather than one that preserves radial information, it is not a perfect indicator of where gravitational pulls on the Local Group are coming from. However, we do expect the direction of the local-structure dipole to feel some influence from the direction of the CMB kinematic dipole.

The observational results for the direction of the dipole are displayed in Fig. 7. The results align with the qualitative expectations detailed above. It turns out that the 2MRS dipole direction is indeed not consistent with the direction of the CMB kinematic dipole, but still within the same basic region of sky.

Note that the constraints on the dipole direction are actually tighter for the very small $0.00 < z < 0.01$ sample than for the full sample of 2MRS galaxies, despite the fact that the number of sources is an order of magnitude smaller. This is not anomalous since the higher redshift sources actually decrease the prominence of the dipole in local structure, producing a result with roughly

half the total S/N as in the case where we take the $z_{\max} = 0.01$ subsample.

The best-fitting direction for the 2MRS dipole is $(l, b) = (228^\circ, 38^\circ)$. Erdogdu et al. (2006) find that the 2MRS number-weighted dipole (the quantity they analyse that is ‘closest’ to our dipole) is at $(l, b) = (218^\circ, 33^\circ)$ in the CMB frame, in close agreement with our results.

4.9 Cutting the supergalactic plane

As a final check, we wish to know how much of the dipole signal in 2MRS is coming from the vicinity of the supergalactic plane (SGP), a planar structure in the local galaxy distribution (Lahav et al. 2000). We therefore progressively excise more and more of the SGP and see how much the amplitude of the dipole dies away. We compare this to the effect of excising *similar areas* from the vicinity of the supergalactic poles. We expect that there should be more sources near the SGP, and that the dipole should die away much more quickly when the SGP is excised than when similar areas around the supergalactic poles are excised.

This check will become more important as we proceed to perform our analysis on surveys that probe much larger radial distances than does 2MRS, as the structure associated with the SGP will only contribute to the dipole on relatively nearby scales, and the effect should diminish as we probe to larger and larger redshifts.

We find the results in Table 4 when we use the usual $|b| < 8^\circ$ Galactic cut and also include a cut around the SGP ($|SGB| \geq$ in the first column⁸) or a cut around the supergalactic poles ($|SGB| <$ in the seventh column). The table is aligned so that cuts around the SGP are in the same row as cuts of similar area around the supergalactic poles. The f_{sky} columns give the fraction of the sky that remains when we perform the given cut. We also calculate f_{sources} , the fraction of the total number N of sources that remain when we perform the given cut. The cuts in SGP (less than $2^\circ, 5^\circ, 10^\circ, 20^\circ$; greater than $74^\circ, 65^\circ, 55^\circ, 41^\circ$) were chosen so that equal areas around the plane and around the poles would be cut if there were no cut in Galactic b (as in our tests on the BATSE catalogue; Section 7). Since there is a cut in Galactic latitude here, f_{sky} does not match up exactly between the cuts around the SGP and supergalactic pole, but the values are still close, and in any case they are normalized in the $f_{\text{sources}}/f_{\text{sky}}$ calculation; we employ the same cuts for all surveys tested in this paper.

From Table 4, we learn the following.

(i) The fraction of sources associated with the SGP is greater than the fraction of sources associated with the supergalactic poles for every f_{sky} . This is our first indication that a greater-than-random portion of the dipole signal comes from the vicinity of the SGP.

(ii) The ratio $f_{\text{sources}}/f_{\text{sky}}$, which gives a measure of how over-dense the uncut portion of the sky is, dwindles steadily as we cut more and more of the SGP, but increases as we cut more and more area around the supergalactic poles. Therefore, as expected, there are more sources near the SGP than near the supergalactic poles, and this is true essentially regardless of how much of the area around the plane/poles is cut.

In general, we conclude that more of the dipole signal comes from the area of the SGP than from the vicinity of the supergalactic poles.

⁸ Supergalactic coordinates SGB and SGL are defined in analogy to Galactic b and l , where SGL is the azimuthal coordinate, and $SGB = 0$ corresponds to the middle of the SGP, so we care only about making cuts in SGB.

Table 4. Key patterns in cutting in supergalactic coordinates for 2MRS.

$ \text{SGB} \geq$	f_{sky}	N	f_{sources}	$\frac{f_{\text{sources}}}{f_{\text{sky}}}$	A_{peak}	$ \text{SGB} <$	f_{sky}	N	f_{sources}	$\frac{f_{\text{sources}}}{f_{\text{sky}}}$	A_{peak}
0.0	0.86	41834	1.00	1.17	0.12	–	–	–	–	–	–
2.0	0.82	39964	0.96	1.16	0.12	74.82	0.84	41234	0.99	1.17	0.12
5.0	0.78	37124	0.89	1.14	0.12	65.90	0.81	39867	0.95	1.18	0.11
10.0	0.70	32673	0.78	1.11	0.12	55.73	0.74	36882	0.88	1.20	0.09
20.0	0.55	24799	0.59	1.08	0.11	41.15	0.59	30321	0.72	1.22	0.05

This serves as a good check that the source of the dipole signal in the relatively local structure surveyed by 2MRS is generally where we expect it to be. When we perform analyses of higher redshift objects in Sections 7 and 8, the SGP should not ‘show up’ as it has here.

4.10 2MRS: conclusion

We conclude our analysis of the dipole signal in 2MRS by pointing out that all results are consonant with theoretical expectations. This comes as no surprise given that 2MRS was the most well controlled and well understood of the surveys we analyse here, and was being treated exhaustively as something of a model for our other analyses.

It should also be noted that we have verified that the results above do not change appreciably when corrections are made for the 2MASS coverage map (see Section 5).

We now proceed to apply our dipole analysis to the full 2MASS data set.

5 DIPOLE IN 2MASS

We now analyse the full 2MASS survey in a manner similar to how we analysed the very well-characterized 2MRS subsample. The challenges associated with analysing 2MASS as a whole are greater, in part because the full sample of 2MASS galaxies (1.6 million extended sources) does not have uniform completeness across the entire sampled sky.

It should be noted that some previous results concerning the 2MASS galaxy distribution stand in some tension with Λ CDM predictions. For example, Frith, Shanks & Outram (2005a) point out that the angular correlation function and angular power spectrum of 2MASS galaxies (under cuts reasonably similar to the ones we perform here) display fluctuations that are 3σ – 5σ out of line with Λ CDM predictions. We focus attention here on the dipole alone, which of course sacrifices a certain amount of information with respect to what could be gained from analysis of the entire power spectrum, but also lends itself to much better and more detailed analysis of contributions to the signal at this one multipole.

5.1 Selection cuts

We make several cuts to the sample of 2MASS galaxies in order to ensure uniformity of the sample as discussed in the following.

(i) As shown in Fig. 8, the biggest issue in connection with survey completeness is that the selection function has a sharp discontinuity for galaxies with K_s -band magnitude greater than roughly 13.5. We therefore cut out all these sources, roughly 2/3 of the sample, at the outset, and consider only that portion of the survey with nearly uniform completeness over the entire sky (with the exception of the Galactic plane), that is, sources with $K_s < 13.5$.

(ii) We must make a more aggressive Galactic cut than we did for 2MRS in order to ensure that star–galaxy confusion does not come into play. Maller et al. (2005) and Skrutskie et al. (2006) note that the 2MASS XSC is highly reliable and complete for $|b| > 20^\circ$ (more than 98 per cent galaxies rather than stars at these latitudes), but that star–galaxy confusion is an increasingly large problem at lower latitudes: the XSC is 10 per cent stars for $5^\circ < |b| < 20^\circ$; and within the Galactic plane, $|b| < 5^\circ$, there is additional contamination by artefacts (10–20 per cent) and Galactic extended sources (~ 40 per cent) including globular clusters, open clusters, planetary nebulae and giant molecular clouds (Jarrett et al. 2000a). In particular, Maller et al. cross-correlate the 2MASS stellar density n_{star} with the XSC galaxy density as a function of the latitude of a symmetric (in Galactic coordinates) cut and find that including XSC objects with $|b| < 15^\circ$ gives a galaxy–star cross-correlation that is higher in amplitude than the galaxy–galaxy autocorrelation, suggesting the presence of multiple-star systems mistakenly identified as extended sources. However, this excess signal goes away for a cut of $|b| < 20^\circ$. Cutting at $|b| < 20^\circ$ ensures less than 2 per cent contamination from Galactic sources (Frith et al. 2005b; Maller et al. 2005).

(iii) We use the XSC confusion flag (cc_flg) to eliminate known artefacts (diffraction spikes, meteor streaks, infrared airglow, etc.).

(iv) Again following Frith et al. (2005b), as well as Maller et al. (2005), we also cut out bright ($K_s < 12.0$) objects with $(J - K_s)$ colours that are outside the range [0.7, 1.4] (see Bilicki 2012 for alternative choices of which ranges to cut). This is a conservative measure designed to get rid of a final set of objects which are in the 2MASS XSC but which are not extragalactic sources. This removes a few thousand sources.

(v) As explained later, we take the 2MASS sky coverage into account. The XSC does not have completely uniform sky coverage given the presence of bright stars and other foreground objects that make it more difficult for the telescopes to detect extended sources in particular directions. Although the pattern of sky coverage is parity-even (following the shape of the Galaxy) and unlikely to mimic a dipole in any way (as implied by the 2MRS analysis), we still take this into account in the present analysis.

(vi) K -corrections (corrections to magnitudes in a given passband that are made necessary by the fact that light can redshift into or out of a given range of wavelengths) for the K_s band can make a non-negligible difference in the calculation of a flux-weighted dipole or other quantity that depends on specifics of photometry. However, in this case we do not need to take them into account because they are actually accounted for in our predictions: K -corrections are tied to the same (pseudo-)Doppler effect that helps to generate the kinematic dipole (see Section 2), and so accounting for them in observational results as well would amount to double-counting.

All photometric cuts are applied to 2MASS isophotal magnitudes – not total magnitudes, which are an extrapolated quantity and viewed as less reliable for the purposes of this kind of analysis. While many analyses which use 2MASS data actually use the

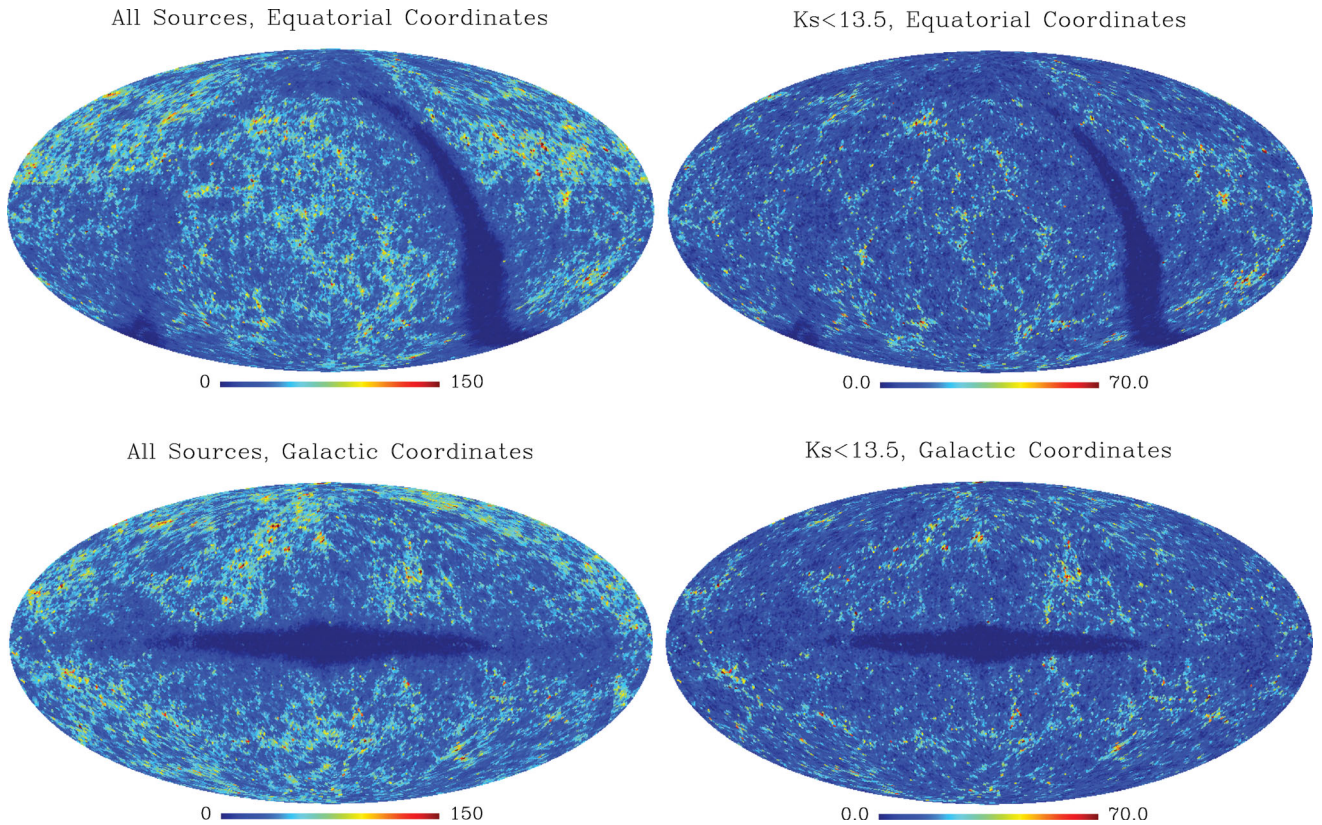


Figure 8. Left-hand side: all 2MASS sources, in equatorial and Galactic coordinates. Note the very strong discontinuity in the selection function, visible in both images (especially in the top image, where it appears as a horizontal line), at declination around 20° . Right-hand side: 2MASS sources with K_s -band magnitude less than 13.5. The survey has nearly uniform completeness when this criterion is imposed.

extrapolated magnitudes (since, according to Jarrett et al. (2003), the isophotal magnitudes underestimate total luminosity by 10 per cent for early-type and 20 per cent for late-type galaxies), we stick here with the more conservative isophotal magnitudes, especially since the cut at $K_s < 13.5$ is much surer to accomplish its purpose if the more conservative magnitude estimates are used. It is worth noting that the 2MRS team used isophotal magnitudes in their sample selection (Huchra et al. 2011).

5.2 Radial selection function

With these photometric cuts applied, we are ready to proceed with the analysis. While no spectroscopic redshifts are available for the 2MASS XSC as a whole, and the photometric redshifts that do exist are not particularly reliable, considerable information is available about the overall radial distribution of 2MASS galaxies. In particular, Frith et al. (2005b) and others give the 2MASS radial selection function as

$$\frac{dN}{dz}(z) = \frac{3z^2}{2(\bar{z}/1.412)^3} \exp\left(-\left(\frac{1.412z}{\bar{z}}\right)^{3/2}\right) \quad (32)$$

with $\bar{z} = 0.074$ for $K_s < 13.5$ and $\bar{z} = 0.050$ for $K_s < 12.5$. With these values of \bar{z} , we can determine theoretical predictions for the local-structure dipole (which is still dominant by two orders of magnitude over other contributions to the dipole at these scales) for these two photometric cuts. Combined with the 2MRS sample, which follows this same form for the selection function quite

closely (see Fig. 9) and corresponds to approximately $\bar{z} = 0.028$, we can perform a comparison of theory and observation for multiple subsamples of the entire 2MASS catalogue.

For $K_s < 12.5$ without (with) the $|b| < 20^\circ$ cut, there are 127 030 (89 980) galaxies. For $K_s < 13.5$ without (with) the cut, there are 542 201 (381 586) galaxies.

5.3 Systematic checks: extinction

In 2MRS, extinction corrections were already applied to the magnitudes of the galaxies, but in 2MASS, the catalogue values for

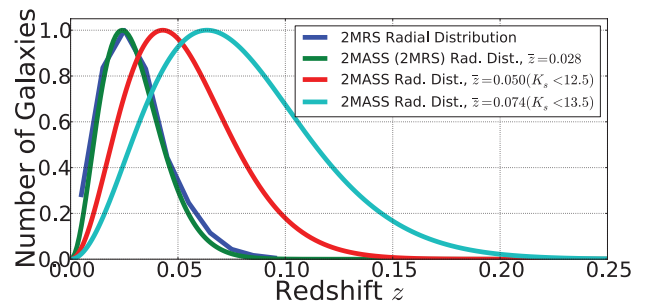


Figure 9. Plot of the radial distribution of 2MASS galaxies as a function of redshift. Three different mean redshifts are shown, one which corresponds to the 2MRS distribution (the actual 2MRS dN/dz is plotted in blue), and the other two of which correspond to photometric cuts in the full 2MASS survey of $K_s < 12.5$ and $K_s < 13.5$.

Table 5. Comparison of dipole parameters without any templates, with SFD template, and with SFD, quadrupole and octopole templates, for 2MASS, for two different limiting K -band magnitudes.

$ b \geq$	Template	NSIDE	N	A_{peak}	l	b	68 per cent CI	95 per cent CI
For objects with $K_s < 13.5$								
20.0	None	128	386 008	0.089	303.4	7.3	0.086–0.092	0.083–0.095
20.0	SFD	128	386 008	0.088	305.0	4.5	0.085–0.091	0.082–0.094
20.0	SFD + Quad + Oct	128	386 008	0.104	268.4	0.0	0.100–0.108	0.096–0.112
For objects with $K_s < 12.5$								
20.0	None	128	91 008	0.0848	275.0	28.2	0.078–0.091	0.072–0.097
20.0	SFD	128	91 008	0.0812	276.3	25.9	0.075–0.088	0.069–0.094
20.0	SFD + Quad + Oct	128	91 008	0.134	267.3	8.5	0.126–0.142	0.117–0.150

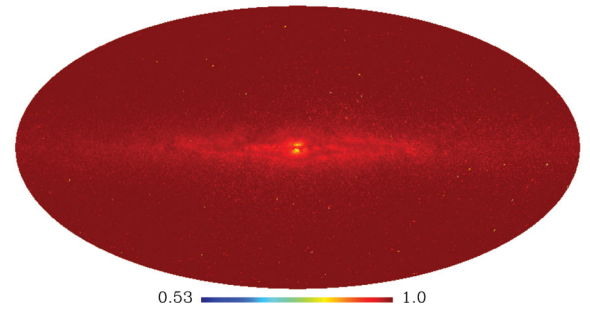
the magnitudes are not corrected for extinction. This means that it becomes much more important in this case to make sure that we have adequately controlled for the effects of extinction. Knowing the magnitudes is important to determine which objects get into the sample in the first place. We find that several thousand galaxies that do not make the $K_s < 13.5$ cut before extinction correction do make the cut when magnitudes are corrected for extinction.⁹

We have performed various extinction corrections, experimenting with slightly different extinction coefficients $R = A_V/E(B - V)$ for the 2MASS K_s band [0.367 from Ho et al. 2008; 0.302 from the analogous United Kingdom Infrared Telescope (UKIRT) value in Schlafly & Finkbeiner 2011; cf. 0.35, which is used by Erdogdu et al. 2006, following Cardelli, Clayton & Mathis 1989]. We find that the results when extinction corrections are applied directly are essentially identical to the results obtained when the SFD dust systematic template is applied as it was to the 2MRS maps, so we explicitly present only the SFD-template results here.

Table 5 shows the effects of the inclusion of the SFD template for the $K_s < 13.5$ and $K_s < 12.5$ maps. As should be clearly visible from a quick glance at the values in the table, very little changes when the SFD template is included, and there is substantial overlap even of the 68 per cent confidence intervals for each of the no-template cases with each of the corresponding SFD-template cases. We conclude that although the results shift slightly, and therefore it is worth keeping the SFD template in our analysis, extinction does not have a substantial impact on the dipole results. This is as expected based on considerations of how extinction affects 2MASS coverage, as outlined in Jarrett et al. (2000b) and Jarrett et al. (2000a): the completeness of the 2MASS XSC is much more adversely affected by source confusion than by extinction.

In the same table, we also show the effect of including templates for the quadrupole and octopole modes, and note that including these templates shifts the peak dipole amplitude by more than 10 per cent for $K_s < 13.5$ and more than 50 per cent for $K_s < 12.5$. (Note also the changes in direction in each case, which actually serve to bring into agreement the dipole directions for the two different photometric cuts.) Based on the results presented in Appendix B, it is not surprising that inclusion of these templates has more of an effect than it did for 2MRS since our sky cut is significantly more aggressive here.

Average Coverage in Each Pixel, Galactic Coordinates

**Figure 10.** 2MASS sky coverage as a function of direction for the entire sky. Coverage can be zero (full masking) or one (no masking), or anywhere between these extremes. We use this map as a systematic template in the dipole formalism.

5.4 Systematic checks: coverage map

Uniform completeness in an infrared survey like 2MASS is impossible due to the presence of foreground stars. In some directions, the presence of foreground stars makes observation of distant background galaxies impossible. Sky coverage, which ranges from 0 to 1 within a given pixel, tends to be well above 0.98 for the high-Galactic-latitude sky. Data products from 2MASS include coverage maps¹⁰ that indicate coverage as a function of direction.

We convert these maps into the same HEALPIX pixelization scheme we use to pixelize all the surveys in this paper, including 2MASS itself. Each HEALPIX pixel contains at least 4, and up to 19, ‘subpixels’ associated with the pixelization of the 2MASS coverage maps, so resolution is not an issue (see Fig. 10).

There are several ways in which we could take these coverage maps into account in our analysis. First, we could mask out all HEALPIX pixels that have an average coverage less than some threshold (the threshold is usually chosen as 0.98 in the literature; see e.g. Ho et al. 2008). Secondly, we could mask out all pixels that have *any* subpixel with coverage less than some threshold. Thirdly, we could use the entire coverage map as a systematic template. We have not found a case in which it makes anything even close to a statistically significant difference which of these strategies we choose, so we choose the option that is simplest and arguably best: we use the entire coverage map as a systematic template. This has

⁹ Note that extinction corrections always bring magnitudes down since sources appear dimmer due to extinction, and so are assigned higher brightness/lower magnitude when corrected for extinction.

¹⁰ See http://www.ipac.caltech.edu/2mass/releases/allsky/doc/sec2_6f.html.

the advantage of not privileging any particular threshold, but rather taking the variation in coverage over the entire sky into account evenhandedly. This accounts for the actual pattern of observations on the sky and weights them accordingly (cf. our treatment of the BATSE exposure function in Section 7). In any case, given that the coverage map closely follows the shape of the Galaxy (foreground stars are, after all, our primary concern), and the Galaxy is nearly parity-even, we do not expect coverage to contribute significantly to the (parity-odd) dipole anyway; results are very much in accord with these expectations.

In summary, we apply the coverage map throughout this section (as we did, in fact, for our 2MRS analysis as well), but find that the results do not change appreciably as a result.

5.5 Systematic checks: sky cut and supergalactic plane

We perform the same cuts in Galactic latitude as we did for 2MRS, but with the expectation that the most reliable results will come for the $|b| < 20^\circ$ cut rather than $|b| < 8^\circ$ as in the case of 2MRS. See Fig. 11 for a visual capture of the results, with cosmic variance on the theoretical prediction taken into account.

The basic conclusion here, as in the case of 2MRS, is that in no case are results outside of the limits expected given cosmic variance. There is tension between the $b_{\text{cut}} = 10^\circ$ sample and other samples for both $K_s < 13.5$ and $K_s < 12.5$, but that is to be expected given the much higher potential for star–galaxy confusion when such low latitudes are included. The same could be said of the $b_{\text{cut}} = 15^\circ$ sample for $K_s < 13.5$. In any case, cosmic variance dominates the error budget for all cases, and dependence of results on Galactic cut indicates no serious contamination from star–galaxy confusion or other systematic effects that vary with Galactic latitude for samples with Galactic $|b| < 20^\circ$, so we follow Skrutskie et al. (2006), Frith et al. (2005b) and others in taking this as our fiducial cut.

We also perform the same test as in the 2MRS case where we cut in supergalactic latitude SGB. We create summary tables giving the key patterns, as we did for 2MRS in Table 4. Results are pre-

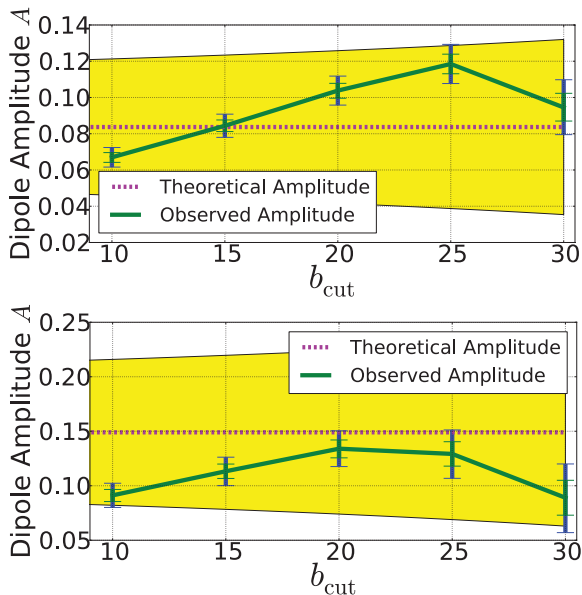


Figure 11. Top panel: dipole amplitude as a function of b_{cut} for 2MASS sources with $K_s < 13.5$. Bottom panel: same for 2MASS sources with $K_s < 12.5$. See Fig. 4 for a fuller discussion of the significance of this type of plot.

sented for $K_s < 13.5$ and $K_s < 12.5$ in Table 6. The observations that we made for 2MRS again (generally) hold here: in particular, $f_{\text{sources}}/f_{\text{sky}}$ decreases monotonically as more and more of the SGP is excised, and the ratio increases (almost) monotonically as more and more of the area around the supergalactic poles is excised.

5.6 Dipole amplitude as a function of redshift/photometric cuts

Taking as most reliable the case with a cut for $|b| < 20^\circ$, and keeping the SFD, quadrupole and octopole templates in place, we proceed to compare theoretical predictions with observational results for the dipole amplitude for the two different magnitude cuts we have used, $K_s < 13.5$ and $K_s < 12.5$, which correspond to $\bar{z} = 0.074$ and 0.050, respectively (where, recall, \bar{z} is defined under equation 32). Again, the 2MRS sample corresponds to $\bar{z} = 0.028$, and we include this data point in our comparisons as well.

See Fig. 12 for results. Note that measurement errors are once again tiny in comparison with cosmic-variance errors.

The magnitude of the dipole in 2MASS has, of course, been calculated previously as part of computations of the entire power spectrum. In particular, Frith et al. (2005b) give $C_1 \approx 0.004$ for both $K_s < 13.5$ and $K_s < 12.5$. This value converts to $A \approx 0.054$; cf. our values of $A = 0.104 \pm 0.004$ and 0.134 ± 0.008 (68 per cent confidence) for $K_s < 13.5$ and $K_s < 12.5$, respectively. While both our measurement and the Frith et al. measurement are in 2σ cosmic-variance agreement with theory, the *mutual* discrepancy between these two observational results is noteworthy. While we do not fully understand this discrepancy with the Frith et al. result, we

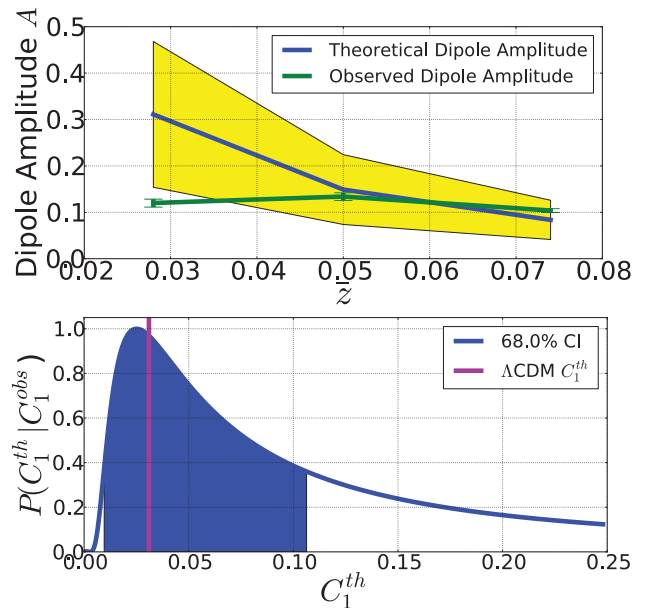


Figure 12. Top panel: results for the dipole amplitude in the 2MASS survey, as a function of mean redshift \bar{z} of the galaxy sample. The $\bar{z} = 0.028$ sample corresponds to the 2MRS galaxies with $|b| < 8^\circ$ cut out; the $\bar{z} = 0.050$ sample to $K_s < 12.5$ with $|b| < 20^\circ$ cut out in the 2MASS XSC; the $\bar{z} = 0.074$ sample to $K_s < 13.5$ with $|b| < 20^\circ$ again cut out in the 2MASS XSC. Cosmic variance (yellow band around the theoretical prediction) is shown in the most pessimistic $|b| < 20^\circ$ case for *all* of the samples. Bottom panel: comparison of the Λ CDM value for C_1^{th} with the expected distribution of C_1^{th} given the observed value for the $K_s < 12.5$ sample. Both panels demonstrate sound agreement between theory and observation for all three subsamples of 2MASS.

Table 6. Comparison of dipole parameters when performing various cuts in supergalactic coordinates, for 2MASS, for two different limiting K -band magnitudes.

$ \text{SGB} \geq$	f_{sky}	N	f_{sources}	$\frac{f_{\text{sources}}}{f_{\text{sky}}}$	A_{peak}	$ \text{SGB} <$	f_{sky}	N	f_{sources}	$\frac{f_{\text{sources}}}{f_{\text{sky}}}$	A_{peak}
For objects with $K_s < 13.5$											
0.0	0.65	386 008	1.00	1.53	0.10	—	—	—	—	—	—
2.0	0.63	368 077	0.95	1.52	0.09	74.82	0.65	385 740	1.00	1.53	0.10
5.0	0.59	342 390	0.89	1.51	0.08	65.90	0.64	378 368	0.98	1.53	0.10
10.0	0.52	301 028	0.78	1.50	0.08	55.73	0.60	355 464	0.92	1.53	0.07
20.0	0.39	225 502	0.58	1.49	0.13	41.15	0.49	295 090	0.76	1.55	0.07
For objects with $K_s < 12.5$											
0.0	0.65	91 008	1.00	1.53	0.13	—	—	—	—	—	—
2.0	0.63	86 657	0.95	1.52	0.11	74.82	0.65	90 951	1.00	1.53	0.13
5.0	0.59	80 177	0.88	1.50	0.10	65.90	0.64	89 203	0.98	1.53	0.12
10.0	0.52	70 212	0.77	1.48	0.10	55.73	0.60	83 805	0.92	1.53	0.10
20.0	0.39	52 528	0.58	1.48	0.14	41.15	0.49	69 923	0.77	1.55	0.10

note that our results are in better agreement if we do *not* marginalize over the dust map and the quadrupole and octopole templates.

5.7 Dipole direction as a function of redshift/photometric cuts

In Fig. 13, we present the results for the dipole direction in both the $K_s < 13.5$ and more conservative $K_s < 12.5$ cases. Once again, dipole amplitudes are of the order of 10^{-1} while the kinematic dipole is expected to be of the order of 10^{-3} , so no particular agreement with the direction of the CMB dipole is expected. We do expect the $K_s < 12.5$ sample to give a direction relatively close to that of the 2MRS dipole, given the overlap in the samples, and $K_s < 13.5$ to give a result close to $K_s < 12.5$. In fact, we would regard it as anomalous if the results were not all consistent with one another, *if* the samples were genuinely sampling the same population – larger samples would simply have smaller error bars than smaller samples. However, the populations being sampled are different, given that the structure associated with $\bar{z} = 0.028, 0.050$ and 0.074 are quite distinct from one another. So even internal inconsistencies (between different values of \bar{z}) are tolerable. Indeed, we find that the direction of the $K_s < 13.5$ dipole is not fully consistent with the direction of the $K_s < 12.5$ dipole, but especially when we include

the quadrupole and octopole modes as systematics templates, as we do in Fig. 13, the inconsistency is very mild.

5.8 2MASS: conclusion

We draw the basic conclusion that there are no anomalous results in applying tests of dipole amplitude and direction to subsets of the 2MASS data set and comparing these results with theoretical predictions. We now proceed to more critical tests using higher redshift objects that might begin to probe the kinematic dipole, as these objects exist at scales on which the local-structure dipole should have become comparable to, or even smaller than, the kinematic dipole.

6 DIFFICULTIES WITH USING X-RAY SURVEYS

We begin by considering surveys that detect very high energy photons (X-ray and gamma-ray), and then address the opposite end of the spectrum (radio).

First, a brief note on X-ray data. Flux-weighted dipoles have been previously calculated using the soft X-ray band (< 2 keV) data from *ROSAT* and the hard X-ray (2–10 keV) background data as observed by HEAO1-A2. For example, Plionis & Georgantopoulos (1999) use the 1.5 keV (~ 0.8 nm) *ROSAT* All-Sky Survey (RASS) data to calculate a flux-weighted dipole, and Scharf et al. (2000) perform a similar analysis on HEAO1-A2. Contamination is a major issue for both analyses. At the time of publication of these studies, only roughly three-quarters of the unresolved X-ray flux in the soft and hard bands had been accounted for (by extrapolation of objects resolved in deep fields) (Scharf et al. 2000), and theoretical modelling of populations contributing to the unresolved flux (active galactic nuclei, starburst galaxies, hot IGM in rich clusters, etc.) remained difficult. In the case of hard X-rays, Scharf et al. argue that at least one-third of the structure in the data may be Galactic in origin (associated especially with the bulge), and soft X-rays are even more strongly contaminated by Galactic emission. Treyer et al. (1998) point out that in the soft band, Galactic emission is present as a contaminant at all scales; the hard band is better, but Treyer et al. still rely on the Galactic hard-band emission model of Iwan et al. (1982) to predict that for Galactic latitude $> 20^\circ$, the variations in flux due to Galactic emission are less than 3 per cent.

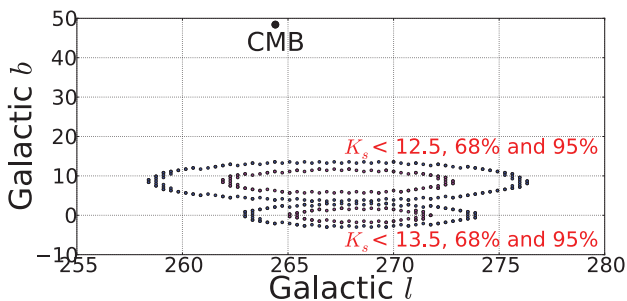


Figure 13. Confidence intervals to go with results for the dipole direction in the 2MASS XSC, $K_s < 13.5$ (smaller circles) and $K_s < 12.5$ (larger circles). In both cases we apply a cut eliminating $|b| < 20^\circ$, and apply the SFD map and quadrupole and octopole maps as systematic templates. The CMB kinematic dipole direction is indicated. Like 2MRS, 2MASS is too shallow to expect agreement between its dipole direction and the direction of the CMB dipole, so this is not an anomalous result.

All told, it is very difficult to remove the foreground in X-ray all-sky surveys successfully without some relatively uncertain modelling, without fairly serious suspicion of contamination, and without removing a good deal of the background too [especially in the soft band; Plionis & Georgantopoulos (1999) estimate that Virgo contributes as much as 20 per cent to the dipole amplitude in RASS]. Theoretical predictions would also be difficult to make without a well-understood redshift distribution, especially given that the populations contributing to the X-ray background are not especially well modelled. Hence we stay away from attempting to perform X-ray analyses here.

Some authors have searched for a dipole in the XRB to the extent that it is possible to do so. The results that Scharf et al. find in hard X-rays for the flux-weighted dipole basically align with theoretical predictions of what they refer to as the Compton–Getting effect [another name for the kinematic dipole, following a paper by Compton & Getting (1935) on the effect on cosmic ray intensity of Earth’s motion through the Milky Way]. More recently, Boughn, Crittenden & Koehrsen (2002) analysed the same (HEAO1-A2) data set and found a limit (95 per cent confidence) on the amplitude of any intrinsic dipole at 5×10^{-3} . However, given the difficulty of definitively separating extragalactic from foreground/Galactic emission in this data set, and other problems already noted, significant uncertainty attends any analysis in X-rays, so we note all these results without making heavy use of them in the remainder of this paper. Similarly, other populations of objects detected at the very high energy end of the spectrum, including blazars and clusters of galaxies detected with gamma-ray satellites (see e.g. Ando et al. 2007), may be good targets for dipole searches in the long term, especially once their bias is better understood and future surveys provide better statistics for the given target population; but we do not pursue those here.

7 DIPOLE IN BATSE GAMMA-RAY BURSTS

For analysis of a 2D-projected dipole (i.e. not flux-weighted; see Section 2) in very high energy surveys, we turn instead to GRBs. GRBs are the most powerful explosions known in the universe, though their exact nature and progenitor objects remain under some debate, and their redshifts are difficult to measure since GRB observations are not well localized (see Fig. 14) and redshifts can only be measured from their afterglows, which must be matched up with the position of the original GRB, a highly non-trivial task given the error bars on the position of a typical GRB. A review of previous research on the dipole in the GRB distribution, as well as presentation of results using the formalism outlined in Section 3 and applied to the BATSE catalogue, is presented in Section 7. In Section 8, we move to the low-frequency end of the spectrum and present results from the NVSS.

7.1 Previous work on the isotropy of GRBs

Up through the mid-1990s, there was a long history of assessing the isotropy of GRBs in an attempt to infer whether they were cosmological or Galactic sources (or a combination thereof). For example, Maoz (1993) argued in 1993 that GRBs could be shown to exist in an extended Galactic halo, some 130–270 kpc away from Earth, by detecting slight but well-defined deviations from spherical symmetry predicted for such a halo population. While his analysis did indeed suggest that GRBs were nearby intergalactic objects, he argued that comparison with more specific models would be necessary before considering the case closed.

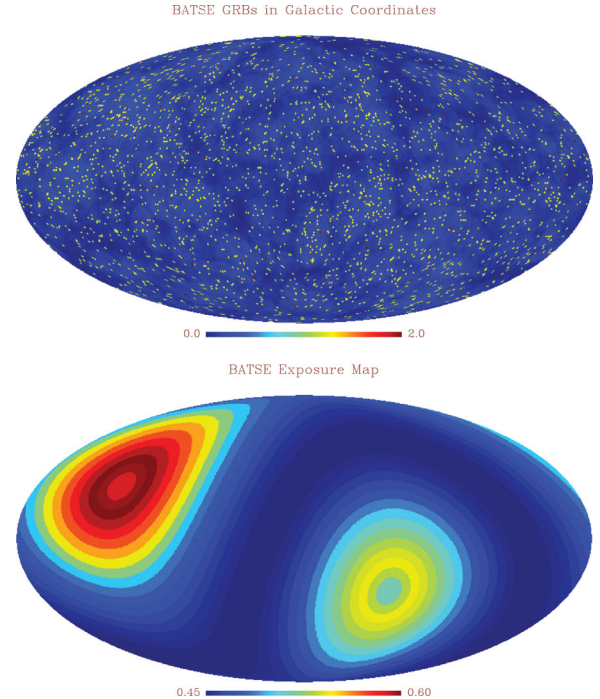


Figure 14. Top panel: GRB positions as recorded by BATSE, with error bars indicated as light circles/ovals around the GRBs; pixellized at NSIDE = 64 (note that the dynamic range is limited to 2 even though a few pixels have 3 or 4 counts in them). Bottom panel: the BATSE exposure function, which is proportional to the amount of time spent monitoring a given direction and varies with declination, in Galactic coordinates.

In a similar spirit, Briggs et al. (1995) argued persuasively that the population of GRBs could not be Galactic, based on their observed isotropy. This study found that the Galactic dipole and quadrupole moments (calculated very straightforwardly as $\langle \cos \theta \rangle$ and $\langle \sin^2 b - 1/3 \rangle$) did not differ significantly from those predicted for an isotropic distribution. The majority of GRB models that assumed GRBs are a Galactic population were found to be in $>2\sigma$ tension with the detected dipole and quadrupole moments, and hence the conclusion of this research was that GRBs are more isotropic than observed Galactic populations, suggesting either a nearby intergalactic or, more likely, cosmological source.

Scharf, Jahoda & Boldt (1995) computed a fluence-weighted dipole (where fluence is flux integrated over the timespan of the burst) in analogy to the flux-weighted dipoles discussed in previous sections of this paper. Combining fluence-weighted dipole information with straightforward 2D-projected dipole measurements (i.e. including photon count information) better distinguishes a velocity dipole (due, as usual, to the Doppler effect and relativistic aberration) from other possible sources of anisotropy. This kind of test can be regarded as a supplement to the kinds of tests we perform here.

The current consensus that GRBs are cosmological is based not only on the considerations discussed above and the absence of even a weak band corresponding to the Milky Way in the GRB distribution (Tegmark et al. 1995), but also (and especially) on the observation of optical, X-ray and radio counterparts to GRBs that are clearly extragalactic (e.g. Metzger et al. 1997; van Paradijs et al. 1997; Fruchter et al. 1999; Paciesas et al. 1999). Given the extragalactic origins of GRBs, we should expect a GRB dipole sourced by the

same effects that give rise to a dipole in other sources we have analysed. As far back as the mid-1990s, Maoz (1994) predicted the dipole in the clustering of GRBs, combining the effects of relativistic aberration and the Doppler effect. These estimates are somewhat uncertain, but we too are unsure how precisely GRBs trace LSS (e.g. if they can be described as a single population with a single bias, and so forth). Maoz finds that the amplitude of this (kinematic) dipole is of the order of $A \sim 10^{-2}$ (to within uncertainties of a factor of 2), which is still an order of magnitude larger than the CMB dipole, but much closer than the sources we dealt with in the previous section. Maoz estimated that a large (of the order of 10^4) sample of GRBs would be necessary to detect the predicted dipole, and given that current catalogues offer only of the order of 10^3 bursts, we do not expect an unequivocal detection. That said, we nevertheless run our tests, given that useful constraints can be placed on the maximum possible dipole amplitude even if we cannot confidently detect a dipole in currently available GRB catalogues.

7.2 BATSE data and systematic effects

We perform our tests on GRBs in the BATSE catalogue, data taken by the *CGRO*. The BATSE instrument onboard *CGRO* detected GRBs within the nominal range of 50–300 keV. Other GRB data sets are available, including those from *Swift* and *Fermi* (known previously as *GLAST*), but we use the BATSE catalogue because it has the most sources for an all-sky survey.

The most recent catalogue of GRBs from BATSE is the ‘current’ catalogue with 2702 sources. In this catalogue, there are several complicating details that are worth noting in regard to what bursts make it into the catalogue and which do not.

BATSE employed scintillators sensitive to gamma-rays from ~ 25 to 2000 keV. A burst triggered the instrument when gamma-ray count rates exceeded some minimum threshold relative to background in two or more of the eight detector modules, within some energy range (Paciesas et al. 1999). Nominally, that energy range was 50–300 keV, but of the order of 30 per cent of BATSE’s observing time was spent with one of several trigger energy ranges different from this. In addition, while the minimum detection threshold in count rates relative to background was 5.5σ as a baseline, this value also changed many times over the course of the experiment, and was not always the same for different time intervals (BATSE tested count rates at 64, 256 and 1024 ms intervals).

A different trigger energy range essentially represents a distinct burst experiment, and a different detection threshold also changes the parameters of the experiment in an important way. However, we argue that the time variation in BATSE’s ability to detect GRBs is not sufficiently great to affect our results on the dipole, especially given the lack of statistics for the BATSE sample of 2702 bursts. Kommers et al. (1997) performed a search for GRBs and other gamma-ray transient phenomena with peak fluxes below (by a factor of ~ 2) the flux necessary to count as a detection, and also with energies outside the nominal 50–300 keV. They found that the direction and intensity distributions of 91 likely GRB candidates *not* included in the final BATSE catalogue imply that biases associated with the trigger mechanism do not significantly affect the completeness of the catalogue. In addition to this result, there is no reason to expect that changes in experimental parameters would have a particular effect on the dipole quantity we investigate here. We note especially that changing trigger criteria and energy ranges do not appreciably increase the chances that GRBs will be confused with Galactic sources [e.g. soft gamma repeaters (SGRs)], so contamination remains minimal; and any changes in trigger cri-

teria apply uniformly over the entire sky, so there is no obvious reason why this would induce a dipole pattern. We therefore proceed to analyse the full catalogue without accounting for these changes.

However, one other experimental parameter is important for our dipole analysis: sky exposure in BATSE varies as a function of declination (BATSE spent different amounts of time looking at different declinations). We create a template out of the exposure function (see Fig. 14) and use this template as one of the systematic templates t_i in the dipole formalism outlined in Section 3. This corresponds to weighting pixels according to how much time the satellite spent observing a given area of the sky: see the approach in, for example, Scharf et al. (1995). The choices we make here, to ignore changes in trigger criteria but take the exposure function into account, corresponds to the choices made in the paper by Tegmark et al. (1995; see also Balazs et al. 1998) calculating the angular power spectrum of the BATSE 3B catalogue, which found no evidence of deviations from isotropy on any angular scale. Our present tests can be regarded as updates (since we use the current catalogue, which more than doubles the number of bursts in the 3B catalogue) of Tegmark’s, with focus on the dipole (direction as well as magnitude) using a real-space estimator that more naturally incorporates sky cuts and systematic templates. (Note that Tegmark et al. also impose a weighted averaging scheme in harmonic space to account for the very large position errors associated with GRBs, which are of the order of degrees, orders of magnitude larger than typical position errors associated with galaxies. This is unnecessary in our case given that the dipole probes scales much larger than the uncertainties in GRB positions.)

The redshift selection function for GRBs is still only poorly understood, though better statistics are consistently being built up. GRBs come from even higher redshifts on average than NVSS sources (see Section 8) (e.g. Xiao & Schaefer 2009), however, so we can confidently say that regardless of the precise distribution, the local-structure dipole will be subdominant in comparison with the kinematic dipole (given that it is already subdominant for the NVSS sources discussed in the next section), and so we consider only the kinematic dipole as a theoretical expectation below.

7.3 Systematic checks and dipole amplitude

The positions of GRBs detected by BATSE are shown in Fig. 14. They are not very well localized; the positions typically have error bars of the order of degrees. The GRBs do not appear to cluster in any particular way by eye, but we apply our usual tests to see whether this holds up rigorously.

In considering what systematic templates to put in place, maps of Galactic foregrounds are unnecessary. In particular, inclusion of the SFD dust template is unnecessary since gamma-rays are highly penetrating and not subject to appreciable dust extinction. We have explicitly verified that the difference between the results including versus not including SFD template is completely negligible. In principle, there is the possibility of confusion with SGRs or other sources of gamma-rays (pulsars, terrestrial gamma-ray flashes, black holes, etc.). This is highly unlikely given that GRBs are easy to distinguish from other gamma-ray sources based on spectral and time-domain data. However, any foreground objects that might contaminate a pure GRB sample are expected to vary with Galactic latitude, and since there is no reason not to do so, we run our usual test of progressively excising the Galactic plane. However, we expect no issues with astrophysical foregrounds given the relatively

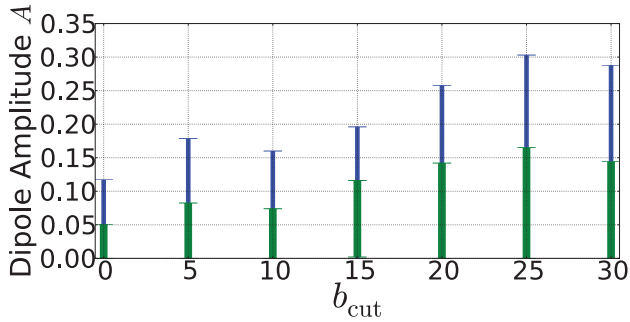


Figure 15. Dipole amplitude in BATSE as a function of b_{cut} . Note that all measurements of the dipole amplitude for different Galactic cuts are mutually consistent, and are consistent with zero as well. Measurement errors in the form of 68 and 95 per cent confidence intervals are shown; they tend to grow with the aggressiveness of the cut, as expected.

clean nature of GRBs as a source; in fact, no sky cut at all should be necessary.

In all tests below, the systematic template we do use, as alluded to above, is the BATSE exposure function, which varies significantly with declination, mimicking (partially) a dipole (see Fig. 14). Some of the results presented below change both quantitatively and qualitatively (e.g. statements we would make about the SGP are different) if this template is not taken into account. We also include quadrupole and octopole templates, as usual. This does not affect the results apart from widening the error bars.

Given that cosmic variance should be much smaller on these scales than it was for 2MASS, we expect that results at different values of minimum $|b|$ will be consistent with each other within the *measurement* error bars. Results for varying Galactic b_{cut} are given in Fig. 15, from which it is clear that results at different b_{cut} are indeed consistent with each other. As expected, there is no detectable signal that varies with Galactic latitude.

We now proceed to run the usual test cutting the SGP. It should be noted that for some cuts, a dipole is detected at marginal significance in this series of tests *if the BATSE exposure function is not taken into account*. Any kind of detection of the SGP in gamma-ray bursts would be very surprising given the complete lack of any association between GRBs and the local structure represented by the SGP (which goes out to something of the order of $z \sim 0.02$ or 0.03 , depending on estimates; see Lahav et al. 2000), but the result turns null when we account for the exposure function. See Table 7 for the results of cutting in supergalactic latitude.

We find another null result, which is good considering that detection of the SGP in GRB data would be a highly unusual find, and would almost certainly indicate something problematic about our analysis.

7.4 Dipole direction and conclusion

The still relatively small number of GRBs (2702) in the BATSE catalogue places only the loosest of constraints on the direction of the dipole. All but a very tiny patch of sky (centred around $(l, b) = (273^\circ, 31^\circ)$, and extending roughly 10° in radius) is within the 3σ confidence interval for the direction of the dipole, and the CMB kinematic dipole direction (which is an appropriate direction of comparison in this case since the kinematic dipole should dominate contributions to the dipole for GRBs) is only marginally outside the 2σ confidence interval (which given the looseness of the constraints is not a noteworthy result).

In all, the BATSE data places useful constraints on the dipole amplitude, but not direction, in GRB data. Our results, while they are not yet strong enough to allow for the detection of the expected kinematic dipole, do place constraints on our ability to distinguish BATSE GRBs as a tracer of LSS, and also constrain any intrinsic dipole in the LSS that would manifest itself in the distribution of GRBs on our sky: at 95 per cent confidence, $A < 0.117$ for the intrinsic dipole. Because the constraints on the dipole are relatively weak relative to constraints using other surveys employed in this paper, and because the radial distribution of BATSE sources $N(z)$ is poorly known, we have not attempted to produce a theoretical expectation for the BATSE local-structure dipole.

In the course of this analysis we have, however, effectively performed several sanity checks on the BATSE data set, showing that the SGP is undetectable (as expected) using our analysis, that the Galaxy does not show up at all in the data (which is in line with previous studies of the GRB distribution), and that the BATSE exposure map must be taken into account in order for these tests not to turn up anomalous. We would of course still like to see constraints on the intrinsic dipole much better than those available from GRBs, those constraints being of the order of 10^{-1} . For this, we turn to the radio survey NVSS.

8 DIPOLE IN NVSS

For a long time, it was assumed that the distribution of radio sources was, like that of GRBs, indistinguishable from isotropic and unclustered (e.g. Webster 1976). In fact, even if the distribution of radio sources was not intrinsically isotropic, radio sources have a large range of intrinsic luminosities, and so structures would naturally wash out when sources were projected on to the sky and radial information was removed (Baleisis et al. 1998). However, recent results, especially using the NVSS, have detected clustering in radio sources, and in particular, a dipole. NVSS is a radio survey with nearly 1.8 million extragalactic sources (Condon et al. 1998). This survey presents an excellent opportunity to actually test for the presence of the kinematic dipole and possibly the intrinsic dipole in LSS. NVSS has more potentially non-negligible systematics to control for than the other surveys we use, but it also has

Table 7. Key patterns in cutting in supergalactic coordinates, for BATSE gamma-ray bursts.

$ \text{SGB} \geq$	f_{sky}	N	f_{sources}	$\frac{f_{\text{sources}}}{f_{\text{sky}}}$	A_{peak}	$ \text{SGB} <$	f_{sky}	N	f_{sources}	$\frac{f_{\text{sources}}}{f_{\text{sky}}}$	A_{peak}
0.0	1.00	2702	1.00	1.00	0.00	–	–	–	–	–	–
2.0	0.97	2606	0.96	1.00	0.00	74.82	0.97	2615	0.97	1.00	0.00
5.0	0.91	2460	0.91	1.00	0.00	65.90	0.91	2479	0.92	1.01	0.00
10.0	0.83	2184	0.81	0.98	0.00	55.73	0.83	2249	0.83	1.01	0.00
20.0	0.66	1740	0.64	0.98	0.00	41.15	0.66	1840	0.68	1.03	0.00

higher potential payoff because of its combination of depth and sky coverage.

8.1 Previous work

Several attempts at calculating the dipole in radio sources have been made in recent years.

Baleisis et al. (1998) present theoretical predictions and observational results for the dipole in the Green Bank 1987 and Parkes–MIT–NRAO (PMN) catalogues. The combination of these catalogues gives $\sim 40\,000$ sources with flux > 50 mJy at 4.85 GHz. They find that the magnitude of the dipole is an order of magnitude larger than expected from the contributions of LSS (analogous to our local-structure dipole) and the kinematic dipole. However, they are plagued by several systematic errors. First, they find that the two catalogues they used have a mismatch in flux. While they correct for this, it is hard to do so with high precision and confidence. They also note that the radio sources in their catalogues are likely drawn from multiple populations, though this is true of any analysis that uses radio sources, and is not a crippling problem if the redshift distribution is sufficiently well understood.

Blake & Wall (2002) attempt to measure the kinematic dipole alone in NVSS (see also Blake, Ferreira & Borrill 2004, for analysis of the rest of the angular power spectrum in NVSS). They make efforts to remove the contribution of what they refer to as the ‘clustering dipole’, the dipole that when flux-weighted gives a measure of the acceleration of the Local Group, and when unweighted matches up with our local-structure dipole. They claim that the clustering dipole should die away by $z < 0.03$ (based on results from the Rowan-Robinson et al. (2000) analysis of the IRAS PSCz dipole, though the results of Erdogdu et al. (2006) call this convergence into question) and contribute roughly $A \sim 2 \times 10^{-3}$ to the total amplitude of the total dipole, if it is not removed as they attempt to do. (Note that we have converted from their peak-to-trough ‘amplitude’ δ to our peak-to-zero amplitude A .)

Blake & Wall measure the remaining dipole – which would ideally be a kinematic dipole only, but which will in reality take contributions not only from local structure beyond what IRAS observed, but also from more distant LSS, as we show below – by expanding the angular distribution of sources in spherical harmonics and measuring the harmonic coefficients $a_{\ell m}$ for the dipole, quadrupole and octopole, including all m values. Inclusion of higher harmonics is necessary because of the lack of full-sky coverage. They find that a dipole model is a good fit by χ^2 , and find good agreement with the direction of the CMB velocity dipole, which they cite as $\theta = 97.2 \pm 0.1^\circ$, $\phi = 168.0 \pm 0.1^\circ$, and which converts to $(l, b) = 264.3, 48.1$.

We take the Blake & Wall results as the most reliable previous result,¹¹ and compare our results to theirs. However, we do have

reason to expect that our estimate of the dipole will be more reliable than theirs; in particular, we do a great deal more to take systematic effects into account than they do in their analysis, using our real-space estimator. We also do not attempt to remove local sources as they do since our objective is to compare our observational results to the full dipole signal expected from theory, which includes both a local-structure and a kinematic contribution. Since we are not flux-weighting, local sources do not contribute preferentially to the dipole, and so we can afford to leave them in the analysis.

8.2 Theoretical predictions

For NVSS, we must be careful to include in our theoretical predictions the contributions of not only the local-structure dipole, but also the kinematic dipole. It is no longer the case, as it was for 2MASS and 2MRS, that the kinematic dipole is swamped by two orders of magnitude by the local-structure dipole. Rather, the two are on the same order of magnitude, as also recognized by Baleisis et al. (1998) and Blake & Wall (2002).

The local-structure contribution is calculated in the same way as always. This part of the prediction will vary somewhat depending on what redshift distribution $n(z)$ we use. Dunlop & Peacock (1990) derived $n(z)$ for several different flux cut-offs, though the results for the dipole amplitude and the redshift distribution itself are somewhat robust to changes in the flux cut-off since radio galaxies display such a wide range of intrinsic luminosities (Baleisis et al. 1998). Blake et al. (2004) note specifically that for the NVSS frequency of 1.4 GHz, the clustering of radio galaxies is not strongly dependent on flux for fluxes between 3 and 50 mJy. The redshift distribution developed by Ho et al. (2008) as a best model for NVSS avoids several drawbacks of the Dunlop & Peacock distribution, especially the assumption that bias is redshift-independent and the heavy reliance on the functional form of the luminosity function rather than data in constraining the redshift distribution. However, without repeating the rather detailed analysis of Ho et al., we are left unable to calculate the redshift distribution for flux cuts different than the 2.5 mJy cut that they use.

Fortunately, we can here make use of the fact that the predictions are not strongly tied to the flux cut-off. We therefore follow Ho et al. in modelling the NVSS redshift distribution as follows:

$$W(z) = \frac{\alpha^{\alpha+1}}{z_*^{\alpha+1} \Gamma(\alpha)} b_{\text{eff}} z^\alpha e^{-\alpha z/z_*}, \quad (33)$$

where $W(z) = b(z)n(z)$, with $b(z)$ being the bias as a function of redshift and $n(z)$ [$\Pi(z)$ in Ho et al.] is the probability distribution for the galaxy redshift. Ho et al. give $b_{\text{eff}} = 1.98$, $z_* = 0.79$ and $\alpha = 1.18$ as best-fitting parameters. Using all of this, we find that for this distribution, the contribution of the local-structure dipole to the total dipole in NVSS is $A = 9.8 \times 10^{-4} \simeq 0.0010$.

Meanwhile, the theoretically expected kinematic dipole may be calculated as shown in Appendix A. The predicted amplitude is

$$A = 2\tilde{\beta} = 2[1 + 1.25x(1 - p)]\beta, \quad (34)$$

where the first term in brackets essentially represents the contribution of relativistic aberration and the second term represents the contribution of the Doppler effect. From the CMB, $\beta = v/c = 1.23 \times 10^{-3}$. Meanwhile, x and p (exponents in the power laws for the intrinsic number counts of galaxies as a function of limiting magnitude, and for the intrinsic flux density of a galaxy as a function of frequency, respectively; see Section 2.3) are not known precisely, but can be estimated, as we do just below.

¹¹ Singal (2011) has performed the most recent analysis in this vein. His results are suspicious, as he finds truly exorbitant speeds for the Local Group (of the order of 1700 km s^{-1}). This becomes more understandable given that the way in which he accounts for the sky cut, particularly the hole in NVSS at declination $< 40^\circ$, is suspect (he simply cuts out declination (Dec.) $> 40^\circ$ as well in order to counterbalance the hole at $< 40^\circ$). Also, his method of detecting the dipole does not account for coupling between the dipole and other multipoles on the cut sky, and also neglects any contribution from the local-structure dipole to the results.

This expression is equivalent to that used by Blake & Wall (2002) (converting from their δ to our $A = \delta/2$):

$$A = [2 + x(1 + \alpha)] \beta, \quad (35)$$

with the substitution in equation (A9) $x \rightarrow 2x/5$ and $p \rightarrow -\alpha$. (The latter substitution is a straightforward matter of notation; the first has to do with switching from magnitudes to fluxes.¹²) We follow Blake & Wall (for NVSS) and also Baleisis et al. (who were not working with NVSS, but who did work with radio catalogues) and take $x \approx 1$ and $\alpha \approx 0.75$, which yields a kinematic dipole amplitude of $A_{\text{kin}} \approx 0.0046$. The full theoretical value includes this kinematic dipole and the contribution from the local-structure dipole ($A_{\text{loc str}} = 0.0010$). We can therefore write the full theoretical prediction as

$$A = 0.0046 \pm 0.0029 \pm 0.0006, \quad (36)$$

where the first uncertainty corresponds to the kinematic dipole and the second to the local-structure dipole. If we find a result that is outside these cosmic-variance errors from the central value, and this is not a systematic effect, we might invoke the presence of an intrinsic dipole as an explanation.

8.3 Present work

We turn first to examining the systematics that need to be accounted for in the NVSS data. These systematics are illustrated in Fig. 16.

First, the survey did not observe below a declination of 40° . For our purposes, the pixellization around this ‘hole’ in declination is especially important to pay attention to, as we find that a sky cut for pixels with centres at Dec. $< 40^\circ$ gives significantly different results for the dipole amplitude and direction than a cut at Dec. $< 37^\circ$ at low-resolution pixellization (NSIDE = 16). We work at much higher resolution (NSIDE = 128), where the effect is not as strong, but we still cut all pixels with Dec. $< 37^\circ$ just to be safe. We choose that particular number following Smith & Huterer (2010), and also because this appears to be a good conservative choice if we want to completely avoid problems associated with the hole in declination.

Secondly, faint sources very close to bright sources cannot be reliably detected. We therefore mask out a 0.6 radius around all sources brighter than 2500 mJy, following Ho et al. (2008). Blake & Wall do not perform this same masking, although they do remove known local sources.

Thirdly, we use as a systematic template the map of Galactic synchrotron radiation of Haslam et al. (1981) and Haslam et al. (1982). This is a 408 MHz radio continuum map of the entire sky that combines data from four different surveys and is dominated by synchrotron emission. Information about foreground emission is important when dealing with a radio survey such as NVSS, and we can apply this map as a systematic template since NVSS could plausibly pick up non-extragalactic signal from this emission.

Fourthly, there is declination-dependent ‘striping’ in the maps, which appears even by eye if no flux cut is imposed. This problem stems from the fact that the NVSS observations were made using

¹² In our original notation, which follows Itoh et al. (2010), the number of galaxies detected by the survey is proportional to $10^{x m_{\text{lim}}}$, where m_{lim} is the limiting magnitude of the survey. In Blake & Wall (2002), the number of galaxies is proportional to $S^{-x'}$, where S is flux, and we have added a prime to distinguish variables named x . Equating $S^{-x'}$ with $10^{x m_{\text{lim}}}$ (up to a proportionality constant) and using the fact that $S \propto 10^{-2 m_{\text{lim}}/5}$, we have that $2 m_{\text{lim}} x'/5 = x m_{\text{lim}}$, or $x = 2x'/5$. Blake & Wall give $x' \approx 1$.

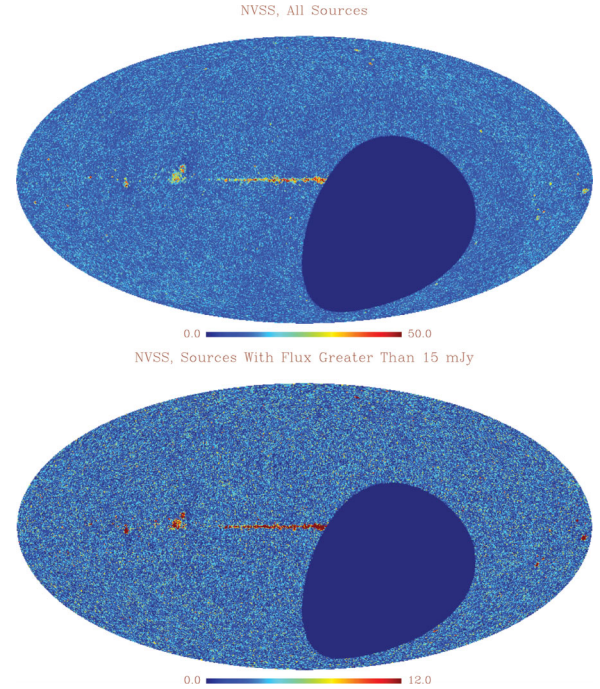


Figure 16. Top panel: all sources in NVSS, in Galactic coordinates. Note the ‘hole’ in the data for declinations less than 40° , and the declination-dependent striping (visible in this coordinate system as a series of ‘wavy’ stripes going outward from the pattern set by the declination-dependent hole in the data). Bottom panel: sources with flux greater than 15 mJy. The spurious power goes largely away with this flux cut, and can no longer be seen by eye. (Dynamic ranges are restricted in both these maps so as to better show structure.)

two different configurations of the Very Large Array (VLA), the D configuration for observations between declinations of -10° and $+78^\circ$, and the DnC configuration for declinations between -40° and -10° , and above $+78^\circ$. The striping is readily apparent by eye for the full catalogue, but is invisible by eye and largely absent even in more rigorous tests for fluxes above ~ 15 mJy (see Blake & Wall 2002). We therefore begin by examining the stability of our dipole results as a function of flux cut.

We find that neither the direction nor the amplitude of the dipole is stable for different flux cuts. This is as expected for flux thresholds less than 15 mJy since the striping artefact gradually dies away as the lower flux threshold is increased from zero up to 15 mJy. However, for flux thresholds above 15 mJy, the fact that the dipole remains unstable is a problem. Increases in the dipole amplitude might be due to the influence of the local-structure dipole, which could come into play more strongly for these brighter (and therefore at least somewhat more local) sources. Fluctuations in the direction (the best-fitting l ranges from 219.1 to 234.3, and the best-fitting b ranges from 11.9 down to -0.2) could also be the result of the local-structure dipole, though this seems unlikely since the b coordinate in particular moves *away* from the direction of the local-structure dipole that we have seen in previous tests using 2MASS as we go to brighter and brighter sources (higher and higher flux thresholds) in NVSS.

We implement two distinct and complementary strategies for dealing with these issues.

- (1) The first strategy is to make a more aggressive cut in declination, which makes use of only one subset of the NVSS maps,

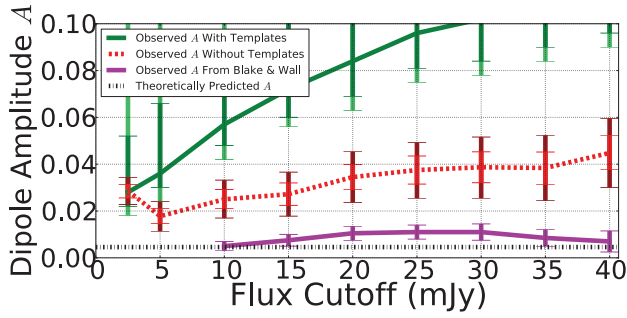


Figure 17. Measured NVSS dipole amplitude as a function of flux cut, with error bars. We show both our results and those from Blake & Wall (2002). For the dotted red curve, we include the Haslam et al. map of 408 MHz Galactic emission as a systematics template, and remove $|b| < 15^\circ$ and Dec. greater than 78° /less than -10° ; for the solid green curve, we include sources with Dec. greater than -37° (thus effectively increasing f_{sky} from 0.42 to 0.60), but also include quadrupole, octopole and declination-dependent-stripping templates. We suspect that the near-monotonic increase of the amplitude with the flux cut is due to spurious power in the NVSS map, and possibly the presence of local structure in the survey as well (since local structures preferentially have higher fluxes). The apparent agreement between theoretical predictions and the Blake & Wall results is partially misleading, as discussed in the text.

corresponding to the D configuration of the VLA: that is, we remove all portions of the sky with declination less than -10° or greater than 78° . These results show a great deal more stability in the direction of the dipole for flux cuts above 5 mJy, and certainly above 15 mJy. The primary results can be seen in Fig. 17. We have also examined the stability of results as a function of cut in Galactic b for the fixed case of a flux cut at 25 mJy. This provides our usual test for contamination that varies as a function of Galactic latitude, and helps justify our choice of $|b| < 15^\circ$ for our cut. We find that there is some fluctuation in the dipole signal as a function of b_{cut} , likely an indication of remaining spurious power in the map rather than genuine Galactic latitude-dependent contamination.

We regard the result with a flux cut at 15 mJy and $|b| < 15^\circ$ as paradigmatic since it provides the best compromise between a large number of sources (getting more sources requires getting into flux ranges where the results are less trustworthy) and having stable results for the dipole. Using this result as the fiducial, we perform our usual test of making cuts in supergalactic latitude, and find that the presence of the SGP is not visible in the results: the results are extremely stable as a function of cut in SGB, with the ratio $f_{\text{sky}}/f_{\text{sources}}$ changing by less than 1 per cent in all but one of the SGB cuts we study. We also compute our likelihood distribution $P(C_1^{\text{th}}|C_1^{\text{obs}})$ using the combination of a 15 mJy flux cut and $|b| < 15^\circ$ cut. We calculate the direction of the dipole for this same case, compare with the CMB kinematic dipole, and show the results in Fig. 18. The final plot of this section is Fig. 19, which depicts the different cuts used in various portions of the analysis for NVSS.

(2) The second strategy for attempting to control the various systematics in NVSS is to include systematics templates that consist of degree-wide stripes in declination: that is, rather than making highly selective cuts in declination as before, we instead keep the entire NVSS map above -37° declination and then include systematics templates that have value 1 within the declination range $(\theta, \theta + 1]$, $\theta = -37^\circ, -36^\circ, \dots, 88^\circ, 89^\circ$, and are zero elsewhere; this is a total of 127 additional templates. This allows us to keep a much greater percentage of the sky in the analysis, and correspond-

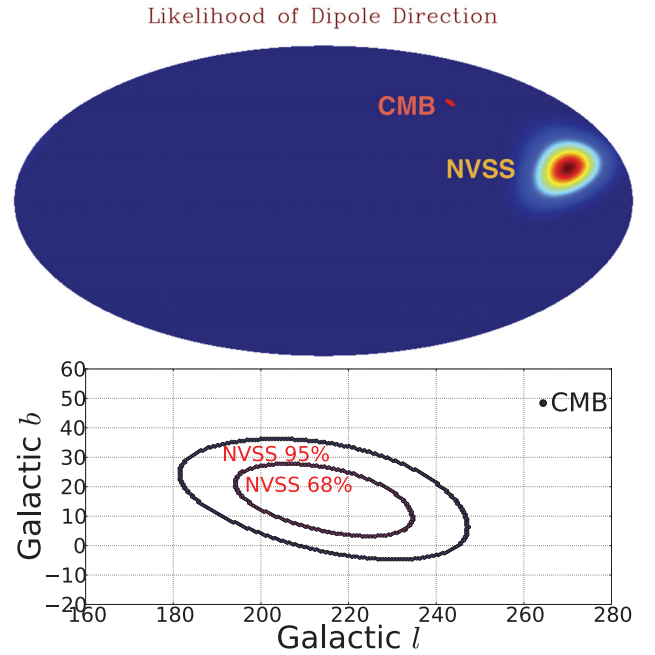


Figure 18. Top panel: likelihood of dipole direction in NVSS, marginalized over amplitude. Bottom panel: the CMB dipole direction is over 3σ away from the best-fitting NVSS dipole direction. This is not a problem, however, since the NVSS dipole amplitude is $A \sim 10^{-2}$, and we expect the LSS dipole direction to match the CMB dipole direction only when amplitudes $A \sim 10^{-3}$ are probed. Both of these plots correspond to the case in which sources have flux greater than 15 mJy, only declinations between -10° and 78° are kept, and $|b| < 15^\circ$ is cut.

ingly more sources. This also allows us to marginalize over the quadrupole and octopole modes by incorporating them as systematics templates too (we avoided doing this with the more aggressive cut in declination since using so many templates on such a small fraction of the sky inflates the dipole amplitudes and error bars). The results of this test are also shown in Fig. 17.

In this case, the single most reliable value for the dipole amplitude is $A = 0.0280$, the value obtained from analysing the sample with flux greater than 2.5 mJy. While the dipole amplitude changes as a function of flux cut, the direction is stable when we keep sources above -37° declination and use templates to correct for the declination-dependent stripping, and we no longer have any clear reason to doubt the results drawn from the most permissive flux cuts. The major caveats in this case are that (a) the use of ‘stripe’ templates may effectively eliminate actual signal as well as spurious power, and (b) the direction we obtain for the dipole in case (2) does not agree with that from case (1).

It is, however, at least somewhat encouraging that the ‘best case’ observed amplitude of the NVSS dipole is similar in both cases: $A = 0.0272$ in case (1) and $A = 0.0280$ in case (2). The agreement between these values may or may not be coincidental, but either way, the value is *not* consistent with the theoretical prediction of $A = 0.0046$, at well over 99 per cent confidence. The reasonable implication is that the dipole we are measuring in the NVSS map is partially spurious. There are several possible sources of this spurious signal: the two most likely are that the declination-dependent stripping may not be wholly taken care of in the maps we test, even with our aggressive flux cuts in case (1) and our templates in case (2) (observational error); or that local sources contribute more strongly to the local-structure dipole than our theoretical modelling allowed

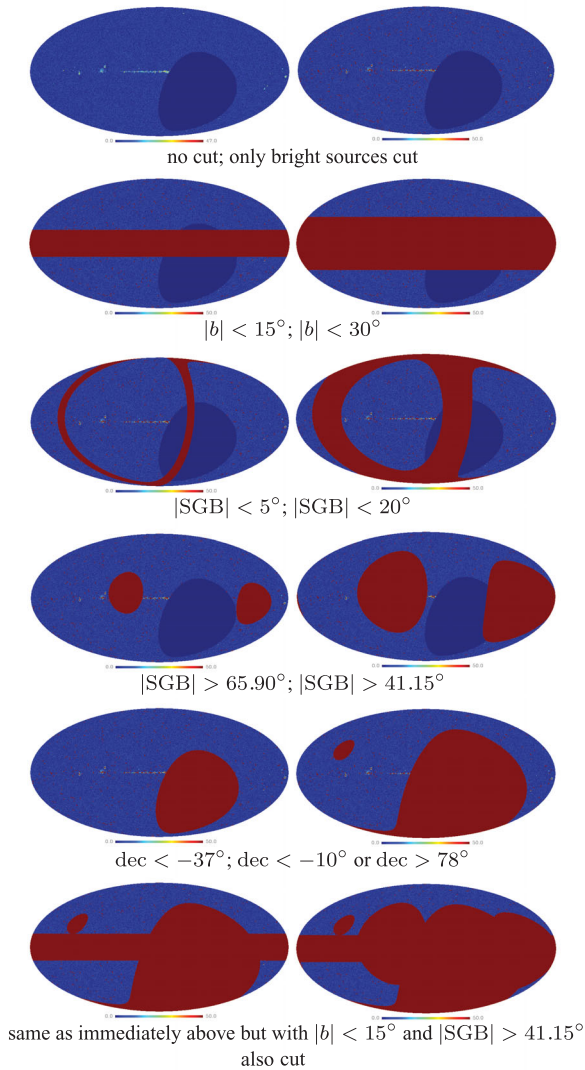


Figure 19. Various cuts employed in the analysis of the NVSS map. The background map is the NVSS map with all the sources that have flux greater than 15 mJy.

for, thus lifting the local-structure dipole’s contribution to the order of 10^{-2} rather than 10^{-3} (theoretical error).

We strongly suspect that observational error plays the much greater role. Theoretical error seems like a reasonable explanation in case (1), given that the theoretical prediction for the dipole amplitude was computed using a flux cut of 2.5 mJy, while the ob-

servational results were computed in case (1) using a flux cut of 15.0 mJy. However, it does not explain the discrepancy in case (2), and in any case the theoretical predictions are ideally fairly robust to flux cuts. (That said, for the most restrictive flux cuts, there is an order-of-magnitude difference in flux between theory and observation, and this is likely to motivate adjusting the theoretical predictions at least somewhat, adding back in the local-structure dipole that we expected to be subdominant.)

As a final note, we also include in Fig. 17 the results that Blake & Wall (2002) found when doing a related dipole analysis on NVSS. There are a handful of reasons why the lack of agreement between our results and theirs is not unexpected. First, they remove local structure in an effort to search for the kinematic dipole only; since we do not do this, our results need not recover theirs, and in fact are expected to give a higher signal. Also, our cut in declination is different than theirs, leading to further potential discrepancies. The apparent agreement between theoretical predictions and the Blake & Wall results is also partially misleading, in that the theoretical prediction includes contributions from both the kinematic dipole and the local-structure dipole at redshifts not excised by Blake & Wall ($z > 0.03$), but they were attempting to measure only the kinematic dipole.

All this said, it is clear that radio surveys of this sort are an excellent setting for the tests we perform, and we look forward to maps from e.g. LOFAR and SKA to perform similar tests.

9 CONCLUSION

In this work, we have focused on what might be called the most straightforward tests of statistical isotropy in LSS – looking for dipole signals in existing surveys over a wide range of wavelengths. It turns out that, despite the relative straightforwardness of the tests themselves (Section 3), the results must be carefully interpreted, as dipole signals take contributions from several different sources. Some of these sources, such as local-structure and kinematic dipoles, are theoretically quite well understood, while others, such as the intrinsic dipole, may be less so; see Section 2.

Observational results in infrared (2MRS/2MASS; Sections 4 and 5), gamma-rays (*CGRO*/BATSE; Section 7) and radio (NVSS; Section 8) turn up no seriously unexpected results, in either dipole amplitude or dipole direction. As long as we are careful to take all sources of dipole signal into account in our theoretical modelling, the observational results are in line with theoretical predictions.

Rigorous tests of this sort, while they ultimately turn up no unexpected results, are valuable tests of current cosmological models, as they add new wavebands in which rigorous tests of statistical isotropy have been conducted, and ensure that statistical isotropy is probed at different epochs, using different surveys with different

Table 8. Summary of most reliable single results from each survey. From left to right in the table appear the name of the survey, the redshift range probed by the survey, the fraction of the sky covered, the number of sources available in the most reliable subset of the data set, the observed dipole amplitude with error bar, the theoretical dipole amplitude (with cosmic-variance error bar if applicable), the direction of the best-fitting observed dipole in Galactic coordinates (l, b), and the most important systematic effect (in some cases, out of several candidates) that must be taken into account in attempting to detect a dipole in the data set.

Survey	Redshift	f_{sky}	N	A_{obs}	A_{th}	$(l, b)_{\text{obs}}$
2MRS	$0 < z < 0.1$	0.86	41 834	0.120 ± 0.009	0.311 ± 0.122	(213°8, 35°2)
2MASS	$0 < z \lesssim 0.2$	0.65	386 008	0.104 ± 0.004	0.084 ± 0.033	(268°4, 0°0)
BATSE	$\bar{z} \gtrsim 2$	1.00	2702	< 0.051 (68 per cent CI)	(unc. prediction)	(weak constraints)
NVSS	$\bar{z} \sim 1$	0.42	211 487	0.027 ± 0.005	0.0046 ± 0.0035	(214°5, 15°6)

systematics. Combined with similar tests using maps of the CMB, these measurements impose interesting constraints on models of, and physical processes during, cosmological inflation. We provide a summary of the most basic results, which are elaborated upon heavily in the body of the paper, in Table 8.

One feature of these results is particularly worth highlighting: namely, that they place constraints on the amplitude of any intrinsic dipole present in LSS. This is especially true of the BATSE and NVSS results since they are not expected to be dominated by the local-structure dipole. BATSE places an upper bound on the intrinsic dipole amplitude at 1×10^{-1} at 95 per cent confidence, while our most optimistic NVSS results places an upper bound at 4×10^{-2} at 95 per cent confidence. As discussed in Section 8.3, however, we cannot place as much confidence in the NVSS result as we do in results from the other three surveys analysed in this paper. Detection of a dipole signal in any survey requires that a great deal of attention be devoted to controlling for systematic errors and spurious power in the survey, and while we have considerable confidence that we have done this successfully for 2MRS, 2MASS and BATSE, we outline in Section 8.3 why acquiring and interpreting NVSS results presents the greatest challenge of all.

Other surveys may provide interesting candidates for these same kinds of tests in the future. Sloan luminous red galaxies are a sufficiently clean data set that these tests may be applicable and workable there (see e.g. Abate & Feldman 2012 for a related test). Also, survey results that have yet to be released may be useful. The *Wide-field Infrared Survey Explorer (WISE)*, which produced a preliminary data release in 2011 April and has full-survey results forthcoming now, covers more than 99 per cent of the sky (Wright et al. 2010). Tests performed on *WISE* would be similar to tests performed on 2MASS, but would update 2MASS results with a more recent and *much* deeper survey. As the X-ray background becomes better understood, this may also serve as an increasingly valuable test of statistical isotropy and setting in which to attempt to detect dipole signals. The Dark Energy Survey will be useful in probing the distribution of galaxies to high redshift, and will have sufficient sky coverage to make the tests presented here useful. In microwaves, dipole signals might be detectable in maps of the gravitational lensing of the CMB, which provide a very good tracer of mass. Finally, new radio surveys such as LOFAR and SKA will probe orders of magnitude more sources down to far lower flux thresholds than NVSS (see e.g. Crawford 2009), and would provide very valuable updates to NVSS results on the dipole amplitude and direction. The kinematic dipole, both direction and amplitude, should be unambiguously detected in these surveys.

ACKNOWLEDGMENTS

For help with the set of research projects that have gone into this paper, thanks to Jim Condon, Sudeep Das, Jerry Fishman, Chris Hirata, Michael Hudson, Tom Jarrett, Lucas Macri, Dominik Schwarz and Weikang Zheng, all of whom were very helpful at different points along the way. For useful comments in the aftermath of initial arXiv posting, we thank Maciej Bilicki and Jim Zibin. CG would additionally like to thank the members of his dissertation committee, all of whom provided valuable input: Philip Hughes, Jeff McMahon, Aaron Pierce, and especially Tim McKay. DH would additionally like to thank Glenn Starkman for sparking his interest in testing statistical isotropy with LSS.

This publication makes use of data products from the Two Micron All Sky Survey, which is a joint project of the University of Massachusetts and the Infrared Processing and Analysis Cen-

ter/California Institute of Technology, funded by the National Aeronautics and Space Administration and the National Science Foundation. We also acknowledge the use of the publicly available HEALPIX package (Górski et al. 2005).

We have been supported by DOE grant under contract DE-FG02-95ER40899, NSF under contract AST-0807564 and NASA under contract NNX09AC89G. DH thanks the Aspen Center for Physics, which is supported by NSF Grant No. 1066293, for hospitality.

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APPENDIX A: RELATIVISTIC ABERRATION AND DOPPLER EFFECT

We first address aberration, following the formalism of Burles & Rappaport (2006) (see also e.g. Calvao et al. 2005), who derive equations for aberration with the ultimate goal of showing that aberration of the CMB temperature might be detectable statistically by *Planck*, looking at shifts of CMB peaks. While this is not our goal, the formalism still holds.

We define a spherical-coordinate system with the z -axis in the direction of motion. If we take the ‘unprimed’ frame to be the CMB frame, and the ‘primed’ frame to be the frame of the Solar system barycentre, then the azimuthal angle ϕ is unchanged between frames: $\phi = \phi'$. However, the polar angle θ is affected as follows:

$$\sin \theta = \frac{\sin \theta'}{\gamma(1 - \beta \cos \theta')}, \quad (\text{A1})$$

where β is the relative velocity of the Solar system with respect to the CMB, $\gamma = (1 - \beta^2)^{-1/2}$ as usual, and $\theta = 0$ corresponds to the direction of forward motion. With the assumption that β is small, which is a good assumption given that CMB observations show it to be of the order of 10^{-3} , expansion in a Taylor series gives

$$\sin \theta = \sin \theta'(1 + \beta \cos \theta'). \quad (\text{A2})$$

Finally, we take the arcsin of both sides and expand the arcsin function assuming small β to obtain

$$\theta = \theta' + \beta \sin \theta'. \quad (\text{A3})$$

We are ultimately interested in calculating how areas (and volumes) on the celestial sphere are stretched or compressed, and hence want the quantity $d\Omega/d\Omega'$. With that in mind, we compute

$$\frac{d\theta}{d\theta'} = 1 + \beta \cos \theta', \quad (\text{A4})$$

$$\frac{\sin \theta d\phi}{\sin \theta' d\phi'} = 1 + \beta \cos \theta', \quad (\text{A5})$$

and find that $d\Omega/d\Omega' = (1 + \beta \cos \theta')^2$. Hence areas and volumes on the sky, proportional to $\sin(\theta)d\theta d\phi$, change as $(1 + \beta \cos \theta')^2 \approx 1 + 2\beta \cos \theta'$.

Itoh et al. (2010) provide a more complete derivation of this, including both the Doppler effect (which changes frequencies and hence measured magnitudes since we never measure bolometric magnitudes) and relativistic aberration, and derive the following expression for the observed angular number density of galaxies $n(\theta)$ given the limiting magnitude m_{lim} :

$$n(\theta, m < m_{\text{lim}}) = \bar{n}(m < m_{\text{lim}})[1 + 2\tilde{\beta} \cos \alpha], \quad (\text{A6})$$

where

$$\tilde{\beta} = [1 + 1.25x(1 - p)]\beta. \quad (\text{A7})$$

Here the intrinsic flux density of a galaxy is assumed to be a power law $S_{\text{rest}}(\nu) \propto \nu^p$, and the intrinsic number counts of galaxies \bar{n} is

$$\bar{n}(m < m_{\text{lim}}) \propto 10^{xm_{\text{lim}}}, \quad (\text{A8})$$

where x is a numerical coefficient of the order of unity. The angle α is the angle between the angular direction θ and the angular direction of the Earth's peculiar velocity \mathbf{v} on the celestial sphere, the same as θ' , but with more convenient notation. The factor of 2 in $2\tilde{\beta}$ above comes from the same source as the square in $(1 + \beta \cos \theta')^2$ earlier. The correction for $\tilde{\beta}$ in equation (A7) is the contribution of the Doppler effect to the overall kinematic dipole in observations of LSS.

The final formula for the combined effects of relativistic aberration and the Doppler effect is $d\Omega/d\Omega' = 1 + 2\beta \cos \theta'$, so that the predicted amplitude of the kinematic dipole is

$$A = 2\tilde{\beta} = 2[1 + 1.25x(1 - p)]\beta. \quad (\text{A9})$$

APPENDIX B: QUADRUPOLE AND OCTOPOLE TEMPLATES

In equation (11) and below, we show how the dipole formalism from Hirata (2009) can be used to separate out genuine dipole signal in a map of LSS from spurious signal due to systematic effects or coupling with the monopole. In this appendix, we detail how we can guard against the possibility that some of the ‘genuine’ dipole signal actually comes from higher multipoles.

The most straightforward way of doing this is simply to include the five $\ell = 2$ and seven $\ell = 3$ modes as systematic templates in the analysis, corresponding to the $\sum_i k_i t_i(\hat{\mathbf{n}})$ term in equation (11). More specifically, the templates $t_i(\hat{\mathbf{n}})$ are assigned to be $Y_{2m}(\hat{\mathbf{n}})$ for $-2 \leq m \leq 2$ and $Y_{3m}(\hat{\mathbf{n}})$ for $-3 \leq m \leq 3$. We expect that quadrupole and octopole modes should not contribute to the dipole signal at all in the limit of full-sky coverage. However, as the sky is cut, multipoles become coupled, and in the limit of very small sky coverage, there is high degeneracy between the quadrupole and octopole modes and the dipole mode, which is the only one of interest.

To quantify this precisely, we run a simple test, the results of which are shown in Fig. B1. We take an artificially generated map that contains nothing but a pure dipole in a certain direction (in this case, $l = 61^\circ 4$, $b = 33^\circ 4$). The dipole amplitude is $A = 0.1$, so we expect that in the full-sky limit, our dipole estimator should recover the result of $A = 0.1$, and the error bars should be the same regardless of whether we include quadrupole and octopole templates – there is, after all, no coupling between the dipole and $\ell = 2/\ell =$

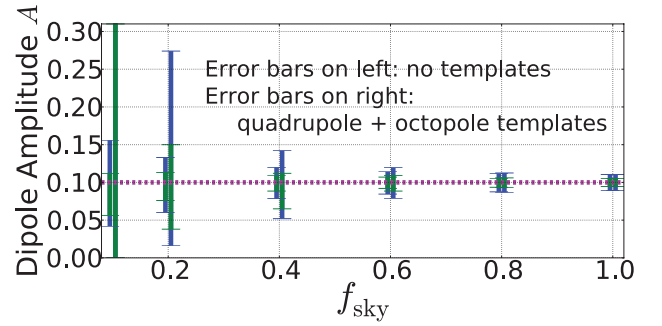


Figure B1. Sensitivity of dipole amplitude measurements on marginalization over the quadrupole and octopole components, as a function of the fraction of the sky covered. We show the dependence of the detected dipole amplitude on f_{sky} for the ideal case where the map being analysed consists of a pure dipole. The dipole has amplitude $A = 0.10$, indicated by the dotted magenta line. We find that while the correct value of the amplitude is, in all cases, recovered within the appropriate error bars, the inclusion of quadrupole and octopole templates does affect the size of the error bars. When f_{sky} is more than about 0.7, the quadrupole and octopole templates make little difference, but below that value of f_{sky} , error bars start increasing noticeably as more of the sky is cut away.

3 modes. However, as we make more and more aggressive cuts (accomplished here by making cuts that are symmetric in Galactic latitude, as we frequently do throughout our analyses of real data sets), we find that including the quadrupole and octopole templates becomes more and more important. While all results are consistent with the correct amplitude of $A = 0.1$, the error bars are much larger when $\ell = 2$ and 3 templates are included when roughly half the sky or more is cut.

Given that the results depend only very weakly on inclusion of quadrupole and octopole templates for small sky cuts, we do not always incorporate quadrupole and octopole templates into our analysis in the limit of nearly full-sky coverage. Meanwhile, especially in the case of NVSS where our analysis deals almost exclusively with less than half the sky, we sometimes explicitly compare the case where quadrupole and octopole templates are not included to the case where they are since inclusion of the templates substantially weakens our results.

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